

Math 200b Winter 2012

Midterm Exam

February 13, 2012

Do as many of the problems as well as you can; the exam may be too long for you to finish. You may use major theorems proved in class or the textbook, but not if the whole point of the problem is reproduce the proof of a theorem proved in class or the textbook. Generally avoid quoting results proved only in homework exercises.

NAME:

Problem 1/ 10	
Problem 2 /10	
Problem 3 /15	
Total	

1. (10 pts)

Let R be an integral domain and let F be its field of fractions. Think of F as an R -module by left multiplication.

Prove that $F \otimes_R F \cong F$, as left F -modules.

2. (10 pts) Give an example of a pair of $n \times n$ matrices A and B , for some $n \geq 1$, such that A and B are not similar, yet A and B have the same characteristic and minimal polynomials. Justify your answer.

3. (15 pts) A module N is called *uniserial* if it has a finite chain of submodules $0 = N_0 \subsetneq N_1 \subsetneq \cdots \subsetneq N_m = N$ such that the N_i are the only submodules of N . For example, a simple module is the same as a uniserial module with $m = 1$.

(a). (10 pts) Suppose that M is a finitely generated torsion module over a PID. Show that $M \cong N_1 \oplus N_2 \oplus \cdots \oplus N_m$ for some uniserial modules $N_1, N_2, \dots, N_m$.

(b). (5 pts) Is the list of uniserial modules N_i in part (a) uniquely determined by M (up to isomorphism and rearrangement, of course)? If yes, prove it, and if not, give a counterexample.