Math 200b Spring 2015 Midterm

February 13, 2015

Do as many of the problems as well as you can; the exam may be too long for you to finish. You may use major theorems proved in class or the textbook, but not if the whole point of the problem is reproduce the proof of a theorem proved in class or the textbook. Simiilarly, quote the result of a homework exercise only if the result of the exercise is a fundamental fact and reproducing the proof of the exercise is not the main point of the problem.

1 (15 pts). Let R be a PID in this problem.

(a) (10 pts). Suppose that M is an R-module. For any ideal I of R, consider the map

$$(i \otimes 1) : I \otimes_R M \to R \otimes_R M$$

obtained by applying $- \otimes_R M$ to the inclusion map $i : I \to R$.

Show that if M is not torsionfree, then there exists a nonzero ideal I of R such that $(i \otimes 1)$ is not injective.

(b) (5 pts). Show that a finitely generated flat R-module M is a free module.

2 (10 pts).

(a) (5 pts). Find all similarity classes of matrices $A \in M_3(\mathbb{Q})$ such that $A^3 = I$.

(b) (5 pts). Find all similarity classes of matrices $A \in M_3(\mathbb{C})$ such that $A^3 = I$.

3 (10 pts).

(a) (5 pts). Let R be an integral domain. Show that if M is a finitely generated R-module which is torsion, then $\mathrm{ann}(M) \neq 0$.

(b) (5 pts). Give an example, with brief justification, of a torsion module M over an integral domain R such that $\mathrm{ann}(M) = 0$.