Math 200a Fall 2014 Exam 1

November 14, 2014

Do as many of the problems as well as you can; the exam may be too long for you to finish. You may use major theorems proved in class or the textbook, but not if the whole point of the problem is reproduce the proof of a theorem proved in class or the textbook. Simiilarly, quote the result of a homework exercise only if the result of the exercise is a fundamental fact and the point of the problem is not to reproduce the result of the exercise.

NAME:

Problem 1/ 15	
Problem 2 /10	
Problem 3 /10	
Total /35	

1. (15 pts)
Let G be a group of order $385 = (5)(7)(11)$.

(a) (3 pts) Show that G has at least 2 Sylow subgroups which are normal.

(b) (5 pts) Find an example of such a group G which is *not* Abelian.

(c) (7 pts) Find a presentation of the group you found in part (b). Give a brief proof that your presentation is correct.

2. (10 pts)
Let P be a Sylow p-subgroup of G, and let $N \trianglelefteq G$.
(a) (5 pts) Show that $P \cap N$ is a Sylow p-subgroup of N.
(b) (5 pts) Show that PN/N is a Sylow p-subgroup of G/N.

3. (10 pts)

(a) (5 pts) Let G be a finite group of order n, and let G act on itself by left multiplication: $g \cdot x = gx$. Consider the homomorphism $\phi : G \to S_n$ associated to this group action. Suppose that $x \in G$ is an element of order d. Prove that the disjoint cycle form of $\phi(x)$ consists of n/d disjoint cycles all of length d.

(b) (5 pts) Let G be a finite group of order $2m$ where m is odd. Again let G act on itself by left multiplication and consider the corresponding homomorphism $\phi : G \to S_{2m}$. Show that $H = \{x \in G \mid \phi(x) \in A_{2m}\}$ is a subgroup of G with $|G : H| = 2$, and that H is the unique subgroup of G of index 2.