Name: ________________________________ **PID:** ____________________

Math 140A: Midterm 2
Foundations of Real Analysis

- You have 50 minutes.
- No books and notes are allowed.
- You may quote any result stated in the textbook or in class.
- You may not use homework problems (without proof) in your solutions.

1. (10 points)

(a) (5 points) Let $\{a_n\}$ be a sequence of positive real numbers such that $\lim_{n\to\infty} a_n = 0$.

Prove that $\lim_{n\to\infty} a_n^p = 0$, for any $p > 0$.

(b) (5 points) Let $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ be sequences of real numbers such that $a_n \leqslant b_n \leqslant c_n$, for all $n \geqslant 1$. Assume that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = l$.

Prove that $\lim_{n\to\infty} b_n = l$.

2. (10 points) Let $\{a_n\}$ and $\{b_n\}$ be bounded sequences of positive real numbers. Prove that $\limsup_{n\to\infty}(a_n b_n) \leqslant (\limsup_{n\to\infty} a_n)(\limsup_{n\to\infty} b_n)$.

3. Let $\{a_n\}$ be a sequence of real numbers such that $\lim_{n\to\infty} a_n = 0$. Define a new sequence $\{b_n\}$ by letting $b_n = a_n - a_{n+1}$, for every $n \geqslant 1$.
Prove that the series $\sum_{n=1}^{\infty} b_n$ is convergent and $\sum_{n=1}^{\infty} b_n = a_1$.

4. (10 points) Let a be a real number.

(a) (5 points) Prove that if $a \geqslant 1$, then the series $\sum_{n=1}^{\infty}(-1)^n \frac{n!}{(a+1)(a+2)...(a+n)}$ is convergent.

(b) (5 points) Prove that if $a \geqslant 2$, then the series $\sum_{n=1}^{\infty}(-1)^n \frac{n!}{(a+1)(a+2)...(a+n)}$ is absolutely convergent.

Do not write on this page.

1		out of 10 points
2		out of 10 points
3		out of 10 points
4		out of 10 points
Total		out of 40 points