Name: ______________________ **PID:** ______________________

Math 140A: Midterm 1
Foundations of Real Analysis

- You have 50 minutes.
- No books and notes are allowed.
- You may quote any result stated in the textbook or in class.
- You may not use homework problems (without proof) in your solutions.

1. (10 points) Let A and B be two nonempty sets of positive real numbers. The "product of A and B" is defined as $C = \{ab | a \in A, b \in B\}$.

Prove that C is bounded below and $\inf C = (\inf A)(\inf B)$.

2. (10 points)

(a) (5 points) Prove that inf $\{\frac{1}{n} \mid n \text{ positive integer}\} = 0$.

(b) (5 points) Prove that if $x, y, z \in \mathbb{R}^k$ (the euclidean k-space), then

$$|x| + |y| + |z| \leqslant |x + y - z| + |x - y + z| + | - x + y + z|.$$

3. (10 points) Let J be the set of all positive integers.

(a) (5 points) Let A be the set of all finite subsets of J. Prove that A is countable.

(b) (5 points) Let B be the set of all subsets of J. Prove that B is uncountable.

4. (10 points) Let X be a metric space with distance function d. Let A be a subset of X and x be a point in X. The "distance from x to A" is defined as $d(x, A) = \inf \{d(x, y) \mid y \in A\}$.

(a) (5 points) Prove that $x \in \bar{A}$ if and only $d(x, A) = 0$.

(b) (5 points) Assume that A is compact. Prove that there exists a point $y \in A$ such that $d(x, y) = d(x, A)$.

Do not write on this page.

1		out of 10 points
2		out of 10 points
3		out of 10 points
4		out of 10 points
Total		out of 40 points