MATH 140A FALL 2015 MIDTERM 2

Instructions: Justify all of your answers, and show your work. You may quote basic theorems proved in the textbook or in class, unless the point of the problem is to reproduce the proof of such a theorem, or the problem tells you not to. Do not quote the results of homework exercises.

1 (10 pts). Decide if the following series converges or not. Justify your answer.

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}}{n^2+5}$$

2. (a) (7 pts). Let $\{p_n\}$ be a sequence in a metric space X. Prove that if $\{p_n\}$ converges, then the sequence is bounded. (This is a result in the text, so you must reproduce the proof here rather quoting that result.)

(b) (3 pts). Give an example of a bounded sequence in a metric space X that does not converge. Briefly justify your answer.

3 (10 pts). Let $E = \{1/n \,|\, n = 1, 2, 3, \dots\} \bigcup \{0\}$ in $\mathbb{R}$. Show that E is compact directly from the definition (not using the Heine-Borel Theorem). In other words, prove directly that any open cover has a finite subcover.

4 (10 pts) Let E and F be compact subsets of a metric space X. Prove that $E \cup F$ is compact.

5 (10 pts). Let $\{p_n\}$ and $\{q_n\}$ be Cauchy sequences in the metric space X with distance function d. Prove that $\lim_{n\to\infty} d(p_n, q_n)$ exists.

Date: November 9, 2015.