# MATH 140A FALL 2015 MIDTERM 1

1. (a) (5 pts). Carefully define the following:

(i). What it means for a set $X$ with a distance function $d$ to be a metric space.

(ii). What it means for $p \in X$ to be a limit point of a subset $E$ of $X$.

(iii). The closure $\overline{E}$ of a subset $E$ of a metric space $X$.

(b) (5 pts). Let $E$ be a nonempty set of real numbers which is bounded below. Prove that $\inf E \in \overline{E}$.

(c) (5 pts). Let $\mathbb{Q}$ be the set of rational numbers in the metric space $\mathbb{R}$. What is $\overline{\mathbb{Q}}$? Justify your answer.

2 (5 pts). Let $\mathbf{x}$ and $\mathbf{y}$ be vectors in the Euclidean space $\mathbb{R}^k$. Prove that

$$|\mathbf{x}+\mathbf{y}|^2 + |\mathbf{x}-\mathbf{y}|^2 = 2|\mathbf{x}|^2 + 2|\mathbf{y}|^2.$$

3. Let $\mathbb{N} = \{1, 2, 3, \dots\}$ be the natural numbers.

(a) (5 pts). Let $A$ be the set of all functions $f : \mathbb{N} \to \{0, 1\}$. Prove that $A$ is uncountable directly by using Cantor's diagonal process (do not quote a theorem from the book).

(b) (5 pts). Let $B$ be the set of all functions $f : \{0, 1\} \to \mathbb{N}$. Is $B$ countable or is it uncountable? Justify your answer.

4. Let $X$ be a metric space. Let $E^\circ$ denote the interior of a subset $E$ of $X$. Suppose that $E$ and $F$ are subsets of $X$.

(a) (5 pts). Is it always true that $E^\circ \cap F^\circ = (E \cap F)^\circ$? Prove or give a counterexample.

(b) (5 pts). Is it always true that $E^\circ \cup F^\circ = (E \cup F)^\circ$? Prove or give a counterexample.

*Date*: October 19, 2015.