

# Math 140a Fall 2015 Homework 8

Due Friday November 20 by 5pm in HW box in basement of AP&M

## Reading

Read Chapter 4. Note that no homework will be due on November 27 (Thanksgiving Holiday)

## Assigned problems from the text (write up and hand in):

Chapter 4: #2, 3, 4, 7, 11, 12, 18, 20

Remarks: In #11, don't do the part that asks you to use the result to give another proof of Exercise 13. In #18, just prove that the function is continuous at every irrational point and not continuous at each rational point. We won't discuss "simple" discontinuities yet. For the second part of #4, first do problem A below.

## Additional problem (write up and hand in)

A. Let  $f : X \rightarrow Y$  and  $g : X \rightarrow Y$  be continuous functions where  $X$  and  $Y$  are metric spaces. Let  $W = \{x \in X \mid f(x) = g(x)\}$ . Prove that  $W$  is a closed subset of  $X$ .

## Optional problem (handing in not required):

B. Chapter 4: #6

Remark: The problem is not so clearly stated. Assume that  $f : E \rightarrow Y$  for some metric space  $Y$ . Then the graph of  $f$  is the subset  $G = \{(x, f(x)) \mid x \in E\}$  of the Cartesian product  $E \times Y$ . The Cartesian product  $E \times Y$  is again a metric space, where  $d((e_1, y_1), (e_2, y_2)) = [d(e_1, e_2)^2 + d(y_1, y_2)^2]^{1/2}$ , so the subset  $G$  is a metric space. Now the problem is to prove that assuming that  $E$  is compact, then  $f$  is continuous if and only if  $G$  is compact.