Math 140a Fall 2015 Homework 5

Due Friday October 30 by 5pm in HW box in basement of AP&M

Reading

All references are to Rudin, 3rd edition.

We will skip the section on perfect sets from Chapter 2 and cover the notion of connectedness from Chapter 2 later in the course. For now, begin to read Chapter 3. This homework has a few more problems on compactness from Chapter 2, and then problems on convergence of sequences and Cauchy sequences from Chapter 3.

Assigned problems (write up full solutions and hand in):

Chapter 2: #15, 17

Chapter 3: #1, 2, 3, 20, 23.

(Hints: For #2, the observation $(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b}) = a-b$ may be helpful. For #3, I suggest first proving by induction that $s_n \leq s_{n+1} < 2$ holds for all $n \geq 1$.)

Additional problem

A. Show that there is a metric space X with a closed and bounded set E which is not compact. Thus the Heine-Borel theorem (Theorem 2.41) does not hold in a general metric space. (Hint: Chapter 2 Exercise 10).

Optional problem (handing in is not required)

Chapter 2: #29