# Math 109 Winter 2010 Homework 9

Due 3/5/10 in class

(All exercise and page numbers refer to Eccles.)

## Reading

Chapters 22, 10, 11, 14. (We will cover only some of the material in chapters 10, 11 and we will not cover those chapters in the same detail as the text. Our main goal is to cover chapter 14 fully.)

## Assigned problems from the text (write up and hand in.)

In the Exercises V which begin on page 271, do
#17(all parts).

## Additional problems not from the text (write up and hand in.)

**1**. Consider the set $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(a, b) | a, b \in \mathbb{R}\}$, that is, the real cartesian plane. Define a relation on $\mathbb{R}^2$ by declaring $(a, b) \sim (c, d)$ if and only if $ab = cd$.

(a) Prove that $\sim$ is an equivalence relation.

(b) Describe geometrically what the equivalence classes of $\sim$ and the corresponding partition of $\mathbb{R}^2$ look like. Sketch some graphs as part of your answer. (Careful! There may be some "special" equivalence classes which don't have the same shape as the others. Make sure you describe those too.)

**2**. Consider the set $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(a, b) | a, b \in \mathbb{R}\}$, that is, the real cartesian plane. Define a relation on $\mathbb{R}^2$ by declaring $(a, b) \sim (c, d)$ if and only if there is some real number $\lambda \neq 0$ such that $(a, b) = (\lambda c, \lambda d)$.

(a) Prove that $\sim$ is an equivalence relation.

(b) Describe geometrically what the equivalence classes of $\sim$ and the corresponding partition of $\mathbb{R}^2$ look like. The same comments hold as for (1b).

In the next problem, you will want to use the concept of prime factorization, which is in Chapter 23 in the book and we probably won't have time to cover fully. The main result you need is that every positive integer $n > 1$ can be expressed as a product of prime powers $n = p_1^{e_i} p_2^{e_2} \dots p_m^{e_m}$ for some distinct primes $p_i$ and exponents $e_i \geq 1$ (we actually did prove this early in the quarter as a consequence of strong induction); and moreover, the factorization is unique up to rearranging the order in which the prime powers are written. (we didn't prove this, although it is not hard with what we know now.) You can make the factorization truly unique if you insist that $p_1 < p_2 < \dots p_m$. For example, $120 = (2^3)(3^1)(5^1)$.

**3**. For $m, n \in \mathbb{Z}$, define $m \sim n$ if and only if both $n | m^k$ and $m | n^j$, for some integers $j, k \geq 1$.

(a). Show that $\sim$ is an equivalence relation on $\mathbb{Z}$.

(b). Describe the equivalence classes $[1], [2], [6]$, and $[12]$.

(c). Using your experiments in part (b) as a guide, for any $m \geq 1$ give a characterization of the equivalence class $[m]$. In other words, define a simple rule (easier to understand than the definition of $\sim$) which determines whether or not an integer is in the equivalence class $[m]$. (Hint: As you might guess from the discussion before the proof, the rule has something to do with prime factorization.)