Math 109 Winter 2010 Homework 7

Due 2/19/10 in class

(All exercise and page numbers refer to Eccles.)

Reading

Finish reading Chapters 8-9 if you are behind (the homework problems are entirely on that material this week), and read chapters 19, 21, and 22. We will not cover section 19.3 or Section 21.4 of those chapters, but feel free to read them of course.

Exercises to submit on Friday 2/19

In the Exercises II which begin on page 115 of the text, do #17, 20.

Comments/hints:

II.17: This problem is trying to seduce you into falling into a trap. Beware. Remember that a function f has an inverse function if and only if f is bijective.

II.20: Recall that we used a different notation for image and inverse image of subsets of functions than the book. I prefer for you to use the more standard notation to get used to it. So instead of writing $\overrightarrow{f}(A)$ for the image under f of a subset A of the domain, the more standard notation is simply $f(A)$.

Additional problems (write up and hand in)

1. (a). Let A be a set with finitely many elements n. Show that if $f : A \to A$ is an injective function, then f is actually bijective. Give an example which shows that the same result does not hold in general if A is infinite.

(b). Let A be a set with finitely many elements n. Show that if $f : A \to A$ is an surjective function, then f is actually bijective. Give an example which shows that the same result does not hold in general if A is infinite.

2. Let A, B, C be sets. Let $f : A \to B$, $g : B \to C$, and $h : A \to B$ be any functions.

(a). Suppose that $g \circ f$ is injective. Prove that this implies that f is injective.

(b). Prove that if g is injective and $g \circ f = g \circ h$, then $f = h$.

3. A function $f : \mathbb{R} \to \mathbb{R}$ is called *increasing* if $f(a) \leq f(b)$ whenever $a < b$, and *strictly increasing* if $f(a) < f(b)$ whenever $a < b$.

(a). Show by example that an increasing function $f : \mathbb{R} \to \mathbb{R}$ need not necessarily be injective.

(b). Show that a strictly increasing function f is injective. Give an example of a strictly increasing function which is not surjective.

(c). Suppose that f is a strictly increasing function which is also surjective. Then f is bijective by part (b), and thus the inverse function f^{-1} exists. In this case, show that $f^{-1} : \mathbb{R} \to \mathbb{R}$ is also a strictly increasing function.