

Math 109 Winter 2010 Homework 6

Due 2/12/10 in class

(All exercise and page numbers refer to Eccles.)

Reading

Read chapters 8 and 9. Also, read the material on the Cartesian product of sets in Chapter 7 if you haven't already.

Assigned problems from the text (write up and hand in.)

In Exercises IV beginning on page 225: #15(i, ii), 16. (By “solve the linear diophantine equation”, this means write down formulas which give all possible solutions for $m, n \in \mathbb{Z}$.)

In the Exercises II beginning on p. 115: #14, 16, 18, 19.

Comments:

In #14, recall that sometimes the symbol \circ for function composition is omitted; so in the last sentence, $fg(x) = gf(x)$ means $f(g(x)) = g(f(x))$.

In #16, you should justify your answers, but you can argue from visual properties of the graphs of these functions and you don't need to write a formal proof that these functions are or are/not injective, surjective, etc.

Additional problems (write up and hand in.)

1. The alien planet of Qoz issues currency called zoqs. All money is in the form of coins, and there are only two kinds of coins: a 13 zoq coin, and a 7 zoq coin.

(a) If you have to pay a bill of 175 zoqs, describe all possible ways in which you can pay this exact amount with no change.

(b). You have to pay a bill of 50 zoqs. Show that you cannot pay this exact amount in coins, but describe a way in which you can pay this bill if the cashier is also allowed to give you change in coins back. Find the way to do this in which the cashier has to give back the smallest possible number of coins.

2. Let A, B, C be sets. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Prove that if f and g are both injective functions, then $g \circ f$ is also injective.

Give a counterexample to show that the converse of the statement you just proved is false: in fact, give an example where $g \circ f$ is injective but g is not injective.

(Hint: in forming counterexamples like the one requested, it is easiest to pick very small sets A, B, C having just a few elements each.)