Math 109 Winter 2010 Homework 5, due 2/5/10 in class

(All exercise and page numbers refer to Eccles.)

1 Reading and practice

Read Chapters 15-18 of the text, and do as many of the end of chapter exercises as possible.

2 Exercises to submit on Friday 2/5

In the Exercises IV which begin on page 225 of the text, do #1, 2, 6, 7, 8, 10, 13.

Comments/hints:

#7. Suggestion: Suppose that the sequence of numbers appearing in the Euclidean algorithm to calculate $\gcd(u_{n+1}, u_n)$ is $a_0 = u_{n+1}, a_1 = u_n, a_2, a_3$, etc. Find a formula relating the numbers a_i to the Fibonacci numbers u_i and prove the formula (by induction, say.) It will help to calculate an example first, say do $\gcd(u_8, u_7) = \gcd(21, 13)$ so you can see what is happening.

#8. To prove the estimate called Lamé's theorem, you will want to quote the Binet formula (Proposition 5.4.3).

#13. This one is challenging! Follow the provided outline. Since the problem is so long, be careful in your writeup to make the different steps of the proof clear; you might want to divide the proof up into numbered steps. As scratchwork, you might want to calculate $\operatorname{lcm}[6, 20]$ first and follow through some of the parts of what you are trying to prove with $a = 6, b = 20$, so you can better see what those parts are saying in this case. (The end result of this problem, that $\operatorname{lcm}[a, b] \gcd(a, b) = ab$, is a useful basic result.)