Math 109 Winter 2010 Homework 3

Due 1/22/10 in class

Reading

All references will be to the Eccles book. Read Chapters 6-7 (and any earlier chapters you didn't read yet) and do the end of the chapter exercises (do not write up) as you read along. Note that the answers to these are in the back of the book.

Assigned problems from the text (write up and hand in.)

Problems I p. 53: #17, 19, 20

(Remark: #19 uses the following notation. Given integers $m \leq n$ and an expression $f(i)$ depending on an integer i, then

$$\prod_{i=m}^{n} f(i) = f(m) \cdot f(m+1) \cdot \cdots \cdot f(n-1) \cdot f(n).$$

In other words, you plug in all integers i in the range between m and n into the formula $f(i)$ and then take the product of all of the results. As another example, we can write $n! = \prod_{i=1}^{n} i$ for any $n \geq 1$.)

Problems II p. 115: #2, 3, 4.

(Remark: I find drawing Venn diagrams useful for intuition, but I consider these part of scratch-work. Go ahead and submit one as part of problem #2, but the written proof of the statement is the important thing. I also do not find the use of "truth tables", which the author uses in Chapter 6, very helpful in proving facts about sets. Do not submit an argument for #2 using a truth table. In all three of the problems above, I want an argument in words along the lines of the proof of Theorem 6.3.4 in the book.)

Additional problems (write up and hand in.)

Recall that the nth Fibonacci number u_n is defined inductively by putting $u_1 = 1$, $u_2 = 1$, and $u_{n+1} = u_n + u_{n-1}$ for $n \geq 2$. In the following problem, do not use the Binet formula.

1. Prove by induction that u_n is even if 3 divides n and u_n is odd if 3 does not divide n.

(Remark: we (basically) showed in class that an integer n is odd if and only if $n = 2m + 1$ for some integer m. From this it is easy to prove that an even integer plus an odd integer is odd, an even plus an even is even, and an odd plus an odd is even. So just assume these facts in your proof.)

2. Prove that $\sqrt{3}$ is irrational. Follow the example of our proof by contradiction in class that $\sqrt{2}$ is irrational (this may also be found as Theorem 13.2.1 in the text.) As we did in that proof, you may assume that every positive rational number q is equal to $q = a/b$ for positive integers a and b and that there exists such a choice of a and b for which the fraction a/b is in lowest terms, in other words such that there is no integer $d > 1$ which divides both a and b.

(Remark: several of you asked me for more details about this assumption that fractions can be written in lowest terms. If this worries you, you can prove it in the following way. Given a rational number q, among all possible choices of positive numerator a and denominator b such that $q = a/b$, choose one such that a is as small as possible; then with this choice the fraction a/b must be in lowest terms and the proof goes through. The fact that there exists such a smallest a follows from the *least natural number principle*, which says that every subset of the set of positive integers must have a smallest element. We take this as an axiom. Apply it to the set of all positive numerators occurring in fractions which are equal to q.)