Math 109 Winter 10 Midterm Exam 2

February 22, 2010

NAME:

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Problem 2 /10	
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1. (10 pts) Here is a calculation that $\gcd(80, 28) = 4$ using the Euclidean algorithm:

$$80 = (2)(28) + 24$$
$$28 = (1)(24) + 4$$
$$24 = (6)(4) + 0$$

(a). (5 pts) Find a solution with $x, y \in \mathbb{Z}$ to the equation $80x + 28y = 12$. Then find a second solution with $x, y \in \mathbb{Z}$ which is different from the first. Show your work, so it is clear how you found these solutions, but no proof is necessary. (You are not required to find *all* solutions, though you can if you want.)

(b). (5 pts) Find *all* solutions to the equation $80x + 28y = 14$. Prove directly, *without quoting a theorem about linear diophantine equations*, that you have found all possible solutions.

2. (10 pts) Let n be an integer. Prove that if 3 divides n^2, then 3 divides n. Make sure you mention any major theorem you are using in the course of your proof.

3. (10 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be the function $f(x) = |x^3|$ (in other words, $f(x)$ is the absolute value of the cube of x.)

Is f injective? Is f surjective? Though you are welcome to draw a graph for scratchwork, justify your answers with a written argument, *not* using a graph, in this problem.

4. (15 pts) Let A, B be sets and let $f : A \to B$, $g : B \to A$ be functions. Suppose that $g \circ f = I_A$. (here, $I_A : A \to A$ is the identity function defined by $I_A(a) = a$ for all $a \in A$.)

(a) (5pts). Show that f is injective.

(b) (5 pts). Show that g is surjective.

(c) (5 pts). With the same hypotheses as above, show that f does not have to be surjective, by giving an explicit example of sets A, B and functions $f : A \to B$, $g : B \to A$ such that $g \circ f = I_A$, but where f is not surjective.

(d) (optional extra credit problem). Now suppose that $A = B$, and that we have functions $f : A \to A$, $g : A \to A$ such that $g \circ f = I_A$. Must f be surjective now? Either prove that f is surjective or give another example where f is not surjective.

Scratch work page