

Math 109 Winter 10 Midterm Exam

January 25, 2010

NAME:

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1. A *tautology* is a statement involving propositional variables P, Q, \dots which is always true no matter what propositions are substituted for the variables.

Is the statement

$$Q \Rightarrow (P \Rightarrow Q)$$

a tautology? Justify your answer, either with truth tables or by writing out an explanation in words if you prefer.

2. Let a, b, c, d be integers, such that 7 divides $a - b$ and 7 divides $c - d$.

(a) Prove that 7 divides $(a + c) - (b + d)$.

(b) prove that 7 divides $ac - bd$.

3. Let A, B, C be sets.

Show that $(A \cup B) \cap C \subseteq A \cup (B \cap C)$. Prove this from scratch, without quoting any formulas from the book.

Show, however, that it is not true in general that $(A \cup B) \cap C = A \cup (B \cap C)$, by demonstrating an explicit example of sets A , B , and C for which the equality fails.

4. Define a sequence of numbers by the following rules. Set $v_1 = 1$, $v_2 = 3$, and define the rest of the sequence inductively by the rule $v_{n+1} = v_n + 2v_{n-1}$ for all $n \geq 2$. Thus the sequence begins 1, 3, 5, 11, 21, 43, 85, ...

Prove by induction that v_n is an odd number for all $n \geq 1$. You may freely use in your proof basic properties of even and odd integers, things like the sum of two odd integers is even, etc.

(scratch work page)