Math 109 Winter 2015 Homework 9

Due 3/6/15 in HW box in basement of AP&M, by 3pm

Reading

Read Chapters 10-13 and do the end of the chapter exercises (do not write up) as you read along.

Assigned problems from the text (write up and hand in.)

In the Problems VI which begin on page 295 of the text, do #6, 7, 8.

Additional problems (write up and hand in.)

1. Consider the set $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(a,b)|a,b \in \mathbb{R}\}$, that is, the real cartesian plane. Define a relation on $\mathbb{R}^2$ by declaring $(a,b) \sim (c,d)$ if and only if $ab = cd$.

(a). Prove that $\sim$ is an equivalence relation.

(b). Describe geometrically what the equivalence classes of $\sim$ and the corresponding partition of $\mathbb{R}^2$ look like. There may be some special equivalence classes which don't have the same shape as the others, so make sure you describe all of the types of equivalence classes. Sketch some graphs as part of your answer.

2. Consider the set $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(a,b)|a,b \in \mathbb{R}\}$, that is, the real cartesian plane. Define a relation on $\mathbb{R}^2$ by declaring $(a,b) \sim (c,d)$ if and only if there is some real number $\lambda \neq 0$ such that $(a,b) = (\lambda c, \lambda d)$.

(a). Prove that $\sim$ is an equivalence relation.

(b). Describe geometrically what the equivalence classes of $\sim$ and the corresponding partition of $\mathbb{R}^2$ look like. There may be some special equivalence classes which don't have the same shape as the others, so make sure you describe all of the types of equivalence classes. Sketch some graphs as part of your answer.

3. Let E be the set of all positive even integers. We say that $n \in E$ is *E-prime* if n cannot be written as a product of two other integers in E.

(a). Show that $n \in E$ is E-prime if and only if $n = 2k$ where $k \geq 1$ is an odd integer.

(b). Show that every $n \in E$ can be written as a finite product of E-primes.

(c). Show that 60 can be written as a product of E-primes in 2 essentially different ways (not the same after rearrangement).

4. Let p be a prime number. For any integer a with $1 \leq a \leq p-1$, the *order* of a modulo p is the smallest positive integer n such that $a^n \equiv 1 (\bmod p)$. Equivalently, in terms of congruence classes the order of $[a]_p$ is the smallest positive integer n such that $([a]_p)^n = [1]_p$. Given $1 \leq a \leq p-1$, we proved Fermat's little theorem in class, which states that $a^{p-1} \equiv 1 (\bmod p)$. Thus the order of a is always at most $p-1$.

(a). Prove that if n is the order of a, then $n|(p-1)$. (Hint: Write $(p-1) = qn + r$ using the division theorem, with $0 \leq r < n$. Show that $r = 0$.)

(b). In the special case $p = 11$, find the order of a for each $1 \leq a \leq 10$.