Math 109 Winter 2015 Homework 8

Due 2/27/15 in HW box in basement of AP&M, by 3pm

Exam

The second midterm is Wednesday February 25. It will concentrate on the topics we have covered since the first midterm: Chapters 7-9, 15-21, especially the material on homeworks 4-7, though of course you are still responsible for understanding the basic background in Chapters 1-6.

The class is now at a size where everyone can fit in Peterson 104. Everyone should come to that room for the exam, there will be no overflow room. Please bring a blue book.

Reading

Read Chapters 23-24 and do the end of the chapter exercises (do not write up) as you read along.

Assigned problems from the text (write up and hand in.)

In the Problems V which begin on page 271 of the text, do #12, 16, 17.

Additional problems (write up and hand in.)

1. Let $m \geq 1$ be a fixed modulus and consider congruence classes modulo m.

(a). Recall that a congruence class $[a]_m$ is called *invertible* if there exists a congruence class $[b]_m$ such that $[a]_m[b]_m = [1]_m$.

Prove that $[a]_m$ is invertible if and only if $\gcd(a, m) = 1$, and in this case there is a *unique* congruence class $[b]_m$ such that $[a]_m[b]_m = [1]_m$. (Hint: translate this to a problem about linear diophantine equations.)

(b). Given a congruence class $[a]_m$, an *additive inverse* for $[a]_m$ is a congruence class $[b]_m$ such that $[a]_m + [b]_m = [0]_m$. Show that every congruence class has a unique additive inverse.