Math 109 Winter 2015 Homework 5

Due 2/6/15 in HW box in basement of AP&M, by 3pm

Reading

All references will be to the Eccles book. Read Chapters 15-16 and do the end of the chapter exercises (do not write up) as you read along.

Assigned problems from the text (write up and hand in.)

In the Problems II beginning on p.115, do problems 16(i)(ii)(v)(vi), 17, 18, 19, 20.

(Remarks: In #16, you should prove your answer carefully. You may use the fact that every nonnegative real number $a \in \mathbb{R}$ has a nonnegative square root $\sqrt{a}$, and that every $a \in \mathbb{R}$ has a cube root $\sqrt[3]{a}$. You may also use whatever basic properties of the exponential function you need to do part (v).)

Additional problems (write up and hand in)

1. Let A, B, C be sets. Let $f : A \to B$ and $g : B \to C$ be functions.

(a). Prove that if f and g are both injective functions, then $g \circ f$ is also injective.

(b). Prove that if $g \circ f$ is injective, then f is injective. Give an example where $g \circ f$ is injective, but g is not injective.

2. A function $f : \mathbb{R} \to \mathbb{R}$ is called *increasing* if $f(a) \leq f(b)$ whenever $a < b$, and *strictly increasing* if $f(a) < f(b)$ whenever $a < b$.

(a). Give an example of an increasing function $f : \mathbb{R} \to \mathbb{R}$ which is not injective.

(b). Show that a strictly increasing function $f : \mathbb{R} \to \mathbb{R}$ is injective.

(c). Give an example of a strictly increasing function $f : \mathbb{R} \to \mathbb{R}$ which is not surjective.

(d). Suppose that f is a strictly increasing function which is also surjective. Show that the inverse function $f^{-1} : \mathbb{R} \to \mathbb{R}$ exists, and show that f^{-1} is also a strictly increasing function.