

MATH 109 FALL 2016 SAMPLE MIDTERM 2

Instructions: Justify all of your answers, and show your work. You may use the result of one part of a problem in the proof of a later part, even if you do not complete the proof of the earlier part. You may quote basic theorems proved in the textbook or in class, unless the problem says otherwise, or unless reproving the result of the theorem is the point of the problem. Do not quote the results of homework exercises.

1 (10 pts). Let $\mathbb{R}^\times = \mathbb{R} \setminus \{0\}$ be the set of all nonzero real numbers. Consider the function $f : \mathbb{R}^\times \rightarrow \mathbb{R}^\times$ defined by $f(x) = 2/x$.

Prove that f is bijective. Find a formula for its inverse function f^{-1} and justify that your formula is correct.

2 (10 pts). Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions, and consider the composition $h = g \circ f : A \rightarrow C$.

(a). Show that if f and g are both injective, then h is injective.

(a). Show that if h is injective, then f is injective. Give an example showing that g does not have to be injective.

3 (10 pts). Consider the decimal expansions of real numbers x with $0 < x < 1$. For each such x , assume the result that x has a unique decimal expansion of the form $x = .a_0a_1a_2a_3\dots$ with $a_i \in \{0, 1, \dots, 9\}$, (that is, $x = \frac{a_0}{10} + \frac{a_1}{100} + \dots + \frac{a_n}{10^n} + \dots$) and where it is not the case that there is some N such that $a_i = 9$ for all $i \geq N$.

Show that the set $X = \{x \in \mathbb{R} \mid 0 < x < 1\}$ is uncountable. (This is a theorem in the book. You are expected to reprove it here.)

4 (10 pts).

A set of 100 voters are asked about how they plan to vote on three statewide ballot propositions, Propositions A, B, and C. On each proposition, they can vote either yes or no.

In the survey, 40 said they plan to vote yes on A, 50 said they will vote yes on B, and 60 said they will vote yes on C. 30 voters plan to vote for both A and B, 10 plan to vote for both A and C, 5 voters plan to vote yes on all three propositions, and 5 voters plan to vote no on all three propositions.

How many voters are there that are planning to vote yes on B but no on both A and C?