MATH 109 WINTER 2015 MIDTERM 1

Instructions: Justify all of your answers, and show your work. You may use the result of one part of a problem in the proof of a later part, even if you do not complete the proof of the earlier part. You may quote basic theorems proved in the textbook or in class, but do not quote the results of homework exercises.

1 (10 pts). Let $Q(a, b, c)$ be the following, where a, b, c are integers:

$$\text{If } a|b \text{ and } b|c, \text{ then } a|(b + c).$$

(a) (5 pts). Prove that $Q(a, b, c)$ is true for all integers $a, b, c \in \mathbb{Z}$.

(b) (3 pts). Write down the *converse* of $Q(a, b, c)$. Is the converse true for all integers a, b, c? Justify your answer.

(c) (2 pts). Write down the *contrapositive* of $Q(a, b, c)$. Is the contrapositive true for all integers a, b, c? Justify your answer.

2 (10 pts). Let A, B and C be sets.

(a) (5 pts). Show that $(A \cup C) - B \subseteq (A - B) \cup C$.

(b) (5 pts). Does $(A \cup C) - B = (A - B) \cup C$ for all sets A, B, C? Justify your answer.

3 (10 pts). Prove that

$$\prod_{i=2}^{n} \left(1 - \frac{1}{i^2}\right) = \frac{n+1}{2n}$$

for all integers $n \geq 2$. [1]

4 (10 pts). Let $S = \{x \in \mathbb{R} | x > 0 \text{ and } x^2 > 4\}$.

(a) (5 pts). Show that $S = \{x \in \mathbb{R} | x > 2\}$.

(b) (5 pts). Show that S does not have a least element. In other words, there does not exist a number $a \in S$ such that $a \leq b$ for all $b \in S$.

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[1]Here, the symbol $\prod_{i=2}^{n}$ means take the product of all of the values you get by substituting the numbers $i = 2, 3, \ldots, n$ into the expression. We could also write

$$\prod_{i=2}^{n}(1 - \frac{1}{i^2}) = (1 - \frac{1}{2^2})(1 - \frac{1}{3^2}) \ldots (1 - \frac{1}{n^2}).$$