MATH 109 FALL 2016 MIDTERM 1

Instructions: Justify all of your answers, and show your work. You may use the result of one part of a problem in the proof of a later part, even if you do not complete the proof of the earlier part. You may quote basic theorems proved in the textbook or in class, unless the problem tells you not to. Do not quote the results of homework exercises. Clearly label any scratchwork in your bluebook.

1 (5 pts). A *tautology* is a statement involving propositional variables $P, Q, \dots$ which is always true no matter what propositions are substituted for the variables.

Is the statement

$$Q \Rightarrow (P \Rightarrow Q)$$

a tautology? Justify your answer.

2 (5 pts). Prove the following statement.

It is not true that for all $x \in \mathbb{R}$, there exists $y \in \mathbb{R}$ such that $xy = 1$.

3 (10 pts). In this problem you may use the following fact: an integer n is odd if and only if $n = 2m + 1$ for some integer m.

(a) Let m be an integer. Prove that $m^2 + m$ is even.

(b) Let n be an integer. Prove that n is odd if and only if $8|(n^2 - 1)$.

4 (10 pts). Let A, B, C be sets.

(a) Show that $A \cap (B \cup C) \subseteq (A \cap B) \cup C$.

(b) Show that $A \cap (B \cup C) = (A \cap B) \cup C$ if and only if $C \subseteq A$.

5 (10 pts). We define a sequence of numbers by induction as follows. Let $v_1 = 3$, $v_2 = 5$, and define $v_{n+1} = 2v_n + v_{n-1}$ for all $n \geq 2$.

(a) Calculate v_5.

(b) Prove that $2^n \leq v_n \leq 3^n$ for all $n \geq 1$.

Date: October 19, 2016.