

Math 109 Fall 2016 Homework 9, not to be handed in, but  
complete by Friday 12/2/2016

## 1 Reading and practice

Read Chapter 22. Do the end of chapter exercises as you read, and check your work against the answers in the back. These exercises are to test your understanding and they are not to be written up and handed in.

## 2 Exercises to finish by Friday 12/2/2016

### Exercises from the text

In the Problems V which begin on page 271 of the text, do #17.

### Additional problems

(We will do an example in class on Monday 11/28 which is similar to the following two problems, so you might want to wait until you see that.)

1. Consider the set  $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(a, b) | a, b \in \mathbb{R}\}$ , that is, the real cartesian plane. Define a relation on  $\mathbb{R}^2$  by declaring  $(a, b) \sim (c, d)$  if and only if  $ab = cd$ .

(a). Prove that  $\sim$  is an equivalence relation.

(b). Describe geometrically what the equivalence classes of  $\sim$  and the corresponding partition of  $\mathbb{R}^2$  look like. There may be some special equivalence classes which don't have the same shape as the others, so make sure you describe all of the types of equivalence classes. Sketch some graphs as part of your answer.

2. Consider the set  $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(a, b) | a, b \in \mathbb{R}\}$ , that is, the real cartesian plane. Define a relation on  $\mathbb{R}^2$  by declaring  $(a, b) \sim (c, d)$  if and only if there is some real number  $\lambda \neq 0$  such that  $(a, b) = (\lambda c, \lambda d)$ .

(a). Prove that  $\sim$  is an equivalence relation.

(b). Describe geometrically what the equivalence classes of  $\sim$  and the corresponding partition of  $\mathbb{R}^2$  look like. There may be some special equivalence classes which don't have

the same shape as the others, so make sure you describe all of the types of equivalence classes. Sketch some graphs as part of your answer.