

Math 109 Fall 2016 Homework 5, due 10/28/2016 in HW  
boxes in the basement of AP&M by 3 pm

## 1 Reading and practice

Read Chapters 10-13. Do the end of chapter exercises as you read, and check your work against the answers in the back. These exercises are to test your understanding and they are not to be written up and handed in.

## 2 Exercises to submit on Friday 10/28

### 2.1 Exercises from the text

In the Problems II which begin on page 115, do #18, 19, 20.

In the Problems III which begin on page 182, do #1.

### Additional problems (write up and hand in)

1. Let  $A, B, C$  be sets. Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be functions.

- (a). Prove that if  $f$  and  $g$  are both injective functions, then  $g \circ f$  is also injective.
- (b). Prove that if  $g \circ f$  is injective, then  $f$  is injective. Give an example where  $g \circ f$  is injective, but  $g$  is not injective.

2. A function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is called *increasing* if  $f(a) \leq f(b)$  whenever  $a < b$ , and *strictly increasing* if  $f(a) < f(b)$  whenever  $a < b$ .

- (a). Give an example of an increasing function  $f : \mathbb{R} \rightarrow \mathbb{R}$  which is not injective.
- (b). Show that a strictly increasing function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is injective.
- (c). Give an example of a strictly increasing function  $f : \mathbb{R} \rightarrow \mathbb{R}$  which is not surjective.

(d). Suppose that  $f$  is a strictly increasing function which is also surjective. Show that the inverse function  $f^{-1} : \mathbb{R} \rightarrow \mathbb{R}$  exists, and show that  $f^{-1}$  is also a strictly increasing function.

3. Show that if any 14 integers are selected from the set  $S = \{1, 2, 3, \dots, 25\}$ , then there are at least two whose sum is 26.

4. Consider a square whose sides have length 1. Show that if we choose any 5 points in the square, some pair of the points will be at most  $\frac{\sqrt{2}}{2}$  apart in distance.