Math 103b Spring 2014 Sample Exam 1

April 18, 2014

1. For each part, give an example of the requested type, with brief justification, if one exists; or else explain why no such example exists.

(a). A commutative ring R with an ideal I which is prime but not maximal.

(b). A commutative ring R with an ideal I which is maximal but not prime.

(c). A field with 7 elements.

(d). An integral domain of characteristic 6.

2. Let $R = \mathbb{Q}(\sqrt{3}) = \{a + b\sqrt{3} | a, b \in \mathbb{Q}\}$.

(a). Prove that R is a subring of the ring $\mathbb{R}$ of real numbers.

(b). Prove that R is a field.

3. Let F be a commutative ring with unity. Show that F is a field if and only if 0 and F are the only ideals of F. (This was a theorem in class. I want you to reprove it here from scratch).

4. Let $R = \mathbb{Z}[i]$ be the Gaussian integers. Consider the principal ideal $I = \langle 4 + i \rangle$ of R.

Show that the factor ring R/I has exactly 17 elements.