Math 103A Fall 2005 Exam 2

November 9, 2005

NAME:

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Problem 1 (30 points)

1a (10 pts) List all of the subgroups of the cyclic group $\mathbb{Z}_{27}$.

1b (10 pts) How many elements of order 3 are there in the group $\mathbb{Z}_9 \oplus \mathbb{Z}_3$?

1c (10 pts) Show that $\mathbb{Z}_{27}$ is not isomorphic to $\mathbb{Z}_9 \oplus \mathbb{Z}_3$.

Problem 2 (20 points)

2a (10 pts) Write the permutation $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 4 & 1 & 2 & 7 & 6 & 8 & 3 \end{bmatrix}$ in disjoint cycle form.

2b (5 pts) Write α as a product of transpositions (2-cycles). Is $\alpha \in A_8$?

2c (5 pts) What is the order of α?

Problem 3 (15 points)

2d (10 pts) Consider the group S_4 and let $\beta = (1234) \in S_4$. Show that β is not in the center of S_4.

2e (5 pts) Let $H = \langle \beta \rangle$ be the cyclic subgroup of S_4 generated by β. How many distinct left cosets of H in S_4 are there? (Your answer should be an actual number and should not involve symbols.)

Problem 4 (20 points)

In parts (a)-(c) of this problem, you may use without proof the following formulas for the structure of $U(n)$ when n is a prime power: $U(2) \cong \{e\}$, $U(4) \cong \mathbb{Z}_2$, $U(2^m) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_{2^{m-2}}$ when $m \geq 3$, $U(p^m) \cong \mathbb{Z}_{p^m - p^{m-1}}$ when p is an odd prime and $m \geq 1$.

4a (5 pts) Show that $U(55)$ is isomorphic to a direct product of cyclic groups.

4b (5 pts) Show that $U(75)$ is isomorphic to a direct product of cyclic groups.

4c (5 pts) Show that $U(55)$ and $U(75)$ are isomorphic to each other.

4d (5 pts) It is claimed in the formulas on the previous page that $U(4)$ is isomorphic to $\mathbb{Z}_2$. Explain why this must be true.

Problem 5 (15 points)

5a (10 pts) Suppose that G is a nonabelian group with $|G| = 14$, and that $x \in G$ is an element such that $x^7 \neq e$. Find $|x|$.

5b (5 pts) Explain why (i.e. perhaps by quoting a theorem) G must be isomorphic to the dihedral group D_7.