Math 103A Fall 2005 Exam 1

Problem 1 (25 points)

(1a) (15 pts) Let G be a nonempty set with a binary operation, that is, a rule assigning to each pair of elements (a, b) with $a, b \in G$ a new element $ab \in G$. Define what it means for G with this operation to be a group.

(1b) (10 pts) Let G be the set consisting of the 4 elements

$$G = \{Chicago, Houston, Anaheim, SaintLouis\}.$$

Abbreviate these elements as c, h, a, s respectively. Define a binary operation on G using the following table:

	c	**h**	**a**	**s**
c	c	h	a	s
h	h	c	s	a
a	a	s	c	h
s	s	a	h	c

Here, the element written in row A and column B of the table is the product AB. For example, from the table we see that $ha = s$. The binary operation defined above satisfies $(a * b) * c = a * (b * c)$ for all $a, b, c \in G$—*you can assume this fact without proof.*

Now prove that G is a group under the binary operation given above.

Problem 2 (25 points)

(2a) (15 pts) Calculate $\gcd(34, 74)$, using any method you like.

(2b) (10 pts) Find integers $x, y \in \mathbb{Z}$ such that $74x + 34y = 4$.

Problem 3 (25 points)

(3a) (5 pts) Let G be a group, and let $a \in G$. Give the definition of the centralizer $C(a)$.

(3b) (15 pts) Prove that $C(a)$ is a subgroup of G.

(3c) (5 pts) Let G be the group $\mathrm{GL}(2, \mathbb{R})$, and consider the element

$$a = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \in G.$$

Calculate $C(a)$.

Problem 4 (25 points)

In this problem, let G be the group $U(12)$.

(4a) (5 pts) List the elements in G.

(4b) (15 pts) Find all of the cyclic subgroups of G.

(4c) (5 pts) Is G itself a cyclic group?