Math 103a Fall 2012 Homework 7

Due *Wednesday 11/21/2012 in class.*

Note the unusual due date for the homework. In particular, we will have class the Wednesday before Thanksgiving. If you will miss lecture that day, you should get the notes from someone (as usual). Also, the homework boxes will not be accessible the beginning of Thanksgiving week due to construction. You must get your homework directly to one of us. You are welcome to hand it in earlier than Wednesday, for example Tuesday in section.

Reading assignment: Finish reading Chapter 8, and begin to read Chapter 9.

Exercises related to Chapter 8:

1. Let G and H be groups. Show that $G \oplus H$ is abelian if and only if both G and H are abelian.

2. Prove that $\mathbb{Z}_8 \oplus \mathbb{Z}_2$ is not isomorphic to $\mathbb{Z}_4 \oplus \mathbb{Z}_4$.

3. You are given that the group $S_3 \oplus \mathbb{Z}_2$ is isomorphic to one of the following groups: $\mathbb{Z}_{12}, \ \mathbb{Z}_6 \oplus \mathbb{Z}_2, \ A_4, \ $ or D_6. Determine which one it must be by a process of elimination.

4. Let $m > 2$ be an even integer and let $n > 2$ be an odd integer. Find a formula for the number of elements of order 2 in $D_m \oplus D_n$.

5. Find both the number of cyclic subgroups of order 15 and the number of elements of order 15 in the group $\mathbb{Z}_{30} \oplus \mathbb{Z}_{20}$.

6. Is $\mathbb{Z}_{10} \oplus \mathbb{Z}_{12} \oplus \mathbb{Z}_6$ isomorphic to $\mathbb{Z}_{60} \oplus \mathbb{Z}_6 \oplus \mathbb{Z}_2$?

7. Suppose that $\phi : \mathbb{Z}_3 \oplus \mathbb{Z}_5 \to \mathbb{Z}_{15}$ is an isomorphism, and that $\phi([2]_3, [3]_5) = [2]_{15}$. Find the element of $\mathbb{Z}_3 \oplus \mathbb{Z}_5$ which ϕ sends to $[1]_{15}$.

8. Prove that $U(140)$ is isomorphic to $U(144)$.

9. How many subgroups does $U(27)$ have?

10. Show that there is no n such that the units group $U(n)$ has order 14.