Math 103a Fall 2012 Homework 6

Due Friday 11/9/2012 by 4pm in homework box in Basement of AP&M

Warning: I am posting this homework early. If this is the week of Halloween and I am away, make sure you are doing Homework 5 which is due on November 2 and not this one. This one is due the end of the week that I return, on November 9.

Reading assignment: Read Chapter 5, and begin to read Chapter 8.

Exercises related to Chapter 5:

1. Let α and β be the permutations given in "box notation" as

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix} \quad \text{and} \quad \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$$

Write α, β, and $\alpha\beta$ as
(a). Products of disjoint cycles;
(b). Products of 2-cycles.

2. Write the following permutations in disjoint cycle form, and then determine the order of each permutation.
(a). $\alpha = (124)(3451)(25)$
(b). $\gamma = (12)(23)(34)(45)(15)$

3. Determine whether the following permutation is even or odd:
$\alpha = (12)(134)(15247)$.

4. Let $\beta = (123)(145)$. Write β^{99} in disjoint cycle form.

5. In S_n, let α be an r-cycle, β an s-cycle, and γ a t-cycle. Complete the following statements (and justify your answer:)
$\alpha\beta$ is an even permutation if and only if $r + s$ is ....
$\alpha\beta\gamma$ is an even permutation if and only if $r + s + t$ is ....

6. Show that A_8 contains an element of order 15.

7. What is the maximum possible order of an element in A_{10}?

8. How many elements of order 5 does S_7 have?

9. In S_4, find a cyclic subgroup of order 4 and a non-cyclic subgroup of order 4.

10. Prove that (1234) cannot be written as a product of (some number of) 3-cycles.

11. Suppose that H is a subgroup of S_n of odd order. Prove that $H \subseteq A_n$.

12. Show that for $n \geq 3$, $Z(S_n) = \{\epsilon\}$.