Math 103a Fall 2012 Homework 4

Due Friday 10/26/2012 by 4pm in homework box in Basement of AP&M

Reading assignment: Chapters 6-7 of Gallian, which we will cover next. (Chapter 5 will be covered in week 5 of the quarter.) In your reading, for the time being you only need to read the parts of chapter 6 labeled "Definition and examples" and "Properties of Isomorphisms". In Chapter 7, you only need to read "Properties of Cosets" and "Lagrange's theorem and consequences".

Exercises related to Chapter 4 and 6

1. Let G be a cyclic group, generated by an element a with $|a| = 12$.

(i) Find all of the subgroups of G.

(ii) Find the orders of the elements a^8, a^9, and a^{10}.

(iii) Find all of the generators of G.

2. Find a collection of distinct cyclic subgroups $\langle a_1 \rangle, \langle a_2 \rangle, \dots \langle a_n \rangle$ of $\mathbb{Z}_{240}$ with the property that $\langle a_1 \rangle \subseteq \langle a_2 \rangle \subseteq \cdots \subseteq \langle a_n \rangle$ and with n as large as possible.

3. Let m and n be positive integers, and let $\langle m \rangle$ and $\langle n \rangle$ be the cyclic subgroups of $\mathbb{Z}$ (under addition) that they generate. Show that the subgroup $H = \langle m \rangle \cap \langle n \rangle$ is also cyclic, and find a formula for a generator of H.

4. Prove or disprove that the groups $U(20)$ and $U(24)$ are isomorphic.

5. Let $G = \left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \middle| a \in \mathbb{R} \right\}$. This is a group under the operation of matrix multiplication. Convince yourself of this, but you don't need to write up the proof.

Show that G is isomorphic as a group to the group $\mathbb{R}$ of real numbers under addition.

6. Show that $U(9)$ is isomorphic to the group Z_6. Write down an explicit function $\phi : U(9) \to Z_6$ which is an isomorphism. Then find a *different* function $\psi : U(9) \to Z_6$ which is also an isomorphism.