Math 103a Fall 2012 Homework 3

Due Friday 10/19/2012 by 4pm in homework box in Basement of AP&M

Reading assignment: Chapters 3-4 of Gallian.

Exercises related to Chapters 3 and 4

1. Prove Theorem 3.6 in the text, which states that if G is a group and $a \in G$, then the centralizer of a, that is $C(a) = \{x \in G | ax = xa\}$, is a subgroup of G.

2. Let $G = \mathrm{GL}(2, \mathbb{R})$. Find the centralizer in G of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. In other words, find $C(A)$.

3. Consider the elements $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$ from the group $\mathrm{GL}(2, \mathbb{R})$. Find the orders of the three elements A, B, and AB in this group.

4. Suppose that $a \in G$ for a group G and that $|a| = 5$. Prove that $C(a) = C(a^3)$.

5. List the elements of the cyclic subgroups $\langle [12] \rangle$ and $\langle [15] \rangle$ inside the group Z_{18}.
Now suppose that G is any group with an element $a \in G$ such that $|a| = 18$. List the elements of the cyclic subgroups $\langle a^{12} \rangle$ and $\langle a^{15} \rangle$ of G. How are your answers to the two parts of this problem related?

6. Find all of the distinct cyclic subgroups of the group $U(18)$. Prove that $U(18)$ is a cyclic group, and find all generators of the group.

7. Prove that an Abelian group with at least two elements of order 2 must have a subgroup of order 4.

8. Let x belong to a group G. If $x^2 \neq e$, $x^3 \neq e$, and $x^6 = e$, find the order of x.

9. Suppose that G is a group that has exactly eight elements of order 3. How many subgroups of order 3 does G have?

10. Let G be an Abelian group with identity e and let n be some fixed positive integer. Prove that that $H = \{x \in G | x^n = e\}$ is a subgroup of G. Show by example that H may not be a subgroup of G if G is not Abelian.

11. Prove that if G is an Abelian group and a and b are elements of G, then if a and b have finite order, then ab also has finite order. Show by example that this statement is not necessarily true if G is not Abelian.