MATH 100C SPRING 2016 FINAL REVIEW SHEET

Your exam is Thursday June 9 in our usual classroom Solis 109, from 11:30-2:30. The exam is cumulative and covers all of the material from the quarter, but may emphasize material from the second half of the quarter slightly. Please bring a blue book.

Some of the problems on the exam will be similar in style to homework problems. I will also ask you to reproduce the proofs of a sample of key theorems we proved. These will be chosen from among the results I list below. (Note that you are not going to be asked to prove all of these on the test, just some of them.) You are still responsible for understanding the statements of all of the theorems we proved and how to use them, but I will not ask you to prove the other theorems not on this list. So you should internalize the proofs of the results below, study homeworks 1-8, and the material in the text and from lecture.

1. Prove that if $K \subseteq F$ is a field extension and $u \in F$ is an algebraic element, then $K(u) \cong K[x]/\langle f(x) \rangle$ as rings, where $f(x)$ is the minimal polynomial of u over K.

2. Prove that if $K \subseteq E \subseteq F$ and $K \subseteq E$ and $E \subseteq F$ are algebraic extensions, then $K \subseteq F$ is an algebraic extension.

3. Prove that if F is a field with p^n elements, where p is prime, then F is the splitting field over its prime subfield $\mathbb{Z}_p$ of the polynomial $x^{p^n} - x$. Conclude by applying another theorem that there is only one field with p^n elements up to isomorphism.

4. Prove that any finite subgroup of the multiplicative group of a field is cyclic. In your argument, you may apply without proof a lemma about finite abelian groups, but carefully state the result about abelian groups you are using.

Date: June 1, 2016.

5. Prove that if $K \subseteq F$ is a field extension and $\theta \in \text{Gal}(F/K)$, then θ defines a permutation of the set of roots in F of the polynomial $f(x) \in K[x]$.

6. Prove that if $\mathbb{Z}_p \subseteq F$ is a field extension, where F is a finite field of order p^n, then $\text{Gal}(F/\mathbb{Z}_p)$ is cyclic of order n.

7. Assuming the result that cyclotomic polynomials are irreducible over $\mathbb{Q}$, prove that if F is the splitting field of $x^n - 1$ over $\mathbb{Q}$ for some $n \geq 2$, then $\text{Gal}(F/\mathbb{Q})$ is isomorphic to $\mathbb{Z}_n^\times$.

8. Assuming that any 2-cycle and 5-cycle in S_5 generate all of S_5, prove that if a polynomial $f(x) \in Q[x]$ of degree 5 has precisely three real roots, then it is not solvable by radicals.