Math 100c Spring 2016 Homework 8

Due Friday 6/3/2016 by 3pm in HW box in basement of AP&M

Problems from the text (write up full solutions):

Section 7.6 #1, 10

Section 8.4 #1, 7, 8

Problems not from the text (write up full solutions):

A. Give a direct proof that if G is a solvable group, so is any subgroup H of G, following the outline below. (The proof in the text is a different one which uses properties of the commutator subgroup.)

Suppose that G is solvable; thus there is a chain of subgroups of G as follows:

$$N_0 = G \supseteq N_1 \supseteq N_2 \supseteq \cdots \supseteq N_m = \{e\},$$

where N_{i+1} is a normal subgroup of N_i and N_i/N_{i+1} is Abelian, for all $0 \leq i \leq m-1$.

If H is any subgroup of G, then there is a chain of subgroups of H as follows:

$$N_0 \cap H = H \supseteq N_1 \cap H \supseteq N_2 \cap H \supseteq \cdots \supseteq N_m \cap H = \{e\}.$$

Show that $N_{i+1} \cap H$ is normal in $N_i \cap H$, and $(N_i \cap H)/(N_{i+1} \cap H)$ is Abelian, for all $0 \leq i \leq m-1$, and conclude that H is solvable. (Hint: to show that $(N_i \cap H)/(N_{i+1} \cap H)$ is Abelian, find a one-to-one homomorphism from this group to N_i/N_{i+1}.)