Math 100c Spring 2016 Homework 3

Due Friday 4/22/2016 by 3pm in HW box in basement of AP&M

Reading

Read Sections 6.5-6.6. We won't cover section 6.7 in this class, which is usually covered in number theory courses such as our Math 104.

Assigned Problems from the text (write up full solutions):

Section 6.4: #1(c)(d), 2(b)(c), 4, 7, 10, 11, 14, 15.

In problems #1, 2, 4, and 14 (all of the problems where the goal is to find a splitting field F of a polynomial $f(x)$ over $\mathbb{Q}$), also calculate, with proof, the degree $[F : \mathbb{Q}]$ of the splitting field over $\mathbb{Q}$.

(Hint for #2(c): how does the given polynomial differ from $(x+1)^3$?)

Addtional problems not from the text (write up full solutions):

A. Let K be a field. Let $f(x) \in K[x]$ be an irreducible polynomial and let $K \subseteq F$ be a splitting field of $f(x)$ over K. Let $u, v \in F$ be any two roots of $f(x)$. Show that there exists an automorphism $\phi : F \to F$ (that is, an isomorphism from F to itself) such that $\phi(u) = v$ and $\phi(a) = a$ for all $a \in K$. (Hint: Examine the proof of Lemma 6.4.4 in the case that $K = L$ and $E = F$.)

B. Show that the result of problem A does not hold if the polynomial $f(x)$ is not assumed to be irreducible, by finding an explicit counterexample.