Math 100c Spring 2016 Homework 1

Due Friday 4/8/2016 by 3pm in HW box in basement of AP&M

Reading

All references are to Beachy and Blair, 3rd edition.

Read Sections 6.1-6.2 and begin to read 6.3. Review any topics from Math 100b that have come up in the lectures that you feel rusty on.

Assigned Problems from the text (write up full solutions and hand in):

Section 6.1: 3, 5, 8(b)

Section 6.2: 1(d)(e)(f), 2(a), 3, 4, 5, 9

Hints: (As always, these are just suggestions; you are welcome to find a different method).

Suggested outline for 6.2 #1(e): do 1(d) first, then show that $\mathbb{Q}(\sqrt{2} + \sqrt[3]{2})$ is the same as the field $\mathbb{Q}(\sqrt{2}, \sqrt[3]{2})$ of 1(d), as follows. Note that if $\alpha = \sqrt{2} + \sqrt[3]{2}$, then α has degree dividing the degree of $[\mathbb{Q}(\sqrt{2}, \sqrt[3]{2}) : \mathbb{Q}]$, which is 6; so it is enough to show that α does not have degree 1, 2, or 3 over $\mathbb{Q}$. To do this, express $1, \alpha, \alpha^2, \alpha^3$ in terms of the basis over $\mathbb{Q}$ you found for $\mathbb{Q}(\sqrt{2}, \sqrt[3]{2})$ and use this to show that they are linearly independent over $\mathbb{Q}$.

Suggested outline for 6.2 #5: Let L be an extension field of F in which $f(x)$ has a root α (using Kronecker's theorem on p. 275). Then consider $F(\alpha)$ and show that $[F(\alpha) : F] = \deg f$.

Problems not from the text (write up full solutions and hand in):

A. Consider the field extension $\mathbb{Q} \subseteq \mathbb{C}$ and suppose that $F = \mathbb{Q}(\alpha_1, \ldots, \alpha_n)$ where $\alpha_i \in \mathbb{C}$ are elements such that $\alpha_i^2 \in \mathbb{Q}$ for all i. Show that $\sqrt[3]{2} \notin F$.

B. Let $K \subseteq F$ be a field extension and let $\alpha \in F$. Show that if $[K(\alpha) : K]$ is odd, then $K(\alpha) = K(\alpha^2)$.