Math 100a Fall 2009 Homework 6

Due 11/6/09 in class or by 4pm in the HW box on the 6th floor of AP&M

Reading

Reading: 3.5, 3.6, 3.7, 3.8 in Beachy and Blair, 3rd edition.

For a warmup to the homework, along with your reading try some of the lower-numbered exercises in sections 3.4, 3.5, and 3.6.

Assigned Problems

Write up neat solutions to these problems:

Section 3.4: 2, 16, 19

Section 3.5: 13, 16, 19, 20

Problems not from the text (also to be handed in):

1. Prove that every group of order 6 is isomorphic to either Z_6 or to S_3. (Hint: Problems 15, 16, 17 in section 3.3 lay out the main steps for you. Note that the multiplication table for S_3 is given in Table 3.3.3 of the text for your reference.)

2. As we showed in class, the dihedral group D_n is isomorphic to a subgroup H of S_n. Namely, each motion determines a permutation by numbering the positions in the plane where the corners lie and tracking how the corners are moved around: if the motion moves the corner in position i to position j, then the corresponding permutation sends i to j.

In this problem, I just want you to do a computation to get a better feel for how this works: write down the 10 permutations in H in case $n = 5$ (use cycle notation.)

3. Find $Z(D_n)$, the *center* of the dihedral group of order $2n$. The answer is not the same for all n, so calculate the answer for some small n first. For this problem, it is probably most convenient to think of D_n as the set $\{e, a, \ldots, a^{n-1}, b, ab, \ldots, a^{n-1}b\}$, where a is a rotation by $360/n$ degrees (so a has order n), and b is some reflection (so b has order 2).

(Hint: recall that the product in D_n satisfies the rule $ba = a^{-1}b$. This rule can be used to determine all other products in D_n.)

4. A *regular tetrahedron* T is a solid in 3-space with four sides which are equilateral triangles. It has 4 vertices and 6 edges. (See wikipedia for a picture.) Consider the group G of rigid motions of T that take place in 3-space. These are the ways of rotating T in 3-space so that it occupies the same location in space afterwards. (reflections are not allowed because they would have to take place in 4-space.) Show that G is isomorphic to A_4, the alternating subgroup of S_4. (Hint: every rigid motion moves around the corners and so determines a permutation. The set H of all such permutations is a subgroup of S_4 isomorphic to G, using the same idea as we did for the dihedral group. Explain why $H = A_4$.)