MATH 100A FALL 2015 MIDTERM 2

Instructions: Justify all of your answers, and show your work. You may use the result of one part of a problem in the proof of a later part, even if you do not complete the proof of the earlier part. You may quote basic theorems proved in the textbook or in class, unless the point of the problem is to reproduce the proof of such a theorem, or the problem tells you not to. Do not quote the results of homework exercises.

1. Let $\phi : G_1 \to G_2$ be a homomorphism between two groups.

(a) (3 pts). Prove that $\phi(e) = e$. (This is a result in the text, so you must reprove it).

(b) (3 pts). Prove that if $a \in G_1$ has finite order, then $o(\phi(a))$ divides $o(a)$. (This is a result in the text, so you must reprove it).

(c) (4 pts). Suppose that $|G_1| = m$ and $|G_2| = n$, with $\gcd(m, n) = 1$. Prove that $\phi(a) = e$ for all $a \in G_1$.

2. Consider $\alpha = (14532)(251)(53) \in S_5$.

(a) (3 pts). Write α as a product of transpositions. Is $\alpha \in A_5$?

(b) (3 pts). Find $o(\alpha)$.

(c) (2 pts). Write α^{-1} in disjoint cycle form.

(d) (2 pts). Is there $\sigma \in S_5$ such that $\sigma^2 = \alpha$? Justify your answer.

3. (a) (3 pts). Suppose that a is an element of a group G with $o(a) = n$ for some $n \geq 1$. If d is a positive divisor of n, find $o(a^d)$. Justify your answer.

(b) (7 pts). Let G be a group with p^k elements, where p is a prime number and $k \geq 1$. Prove that G has a subgroup H with $|H| = p$.

4 (10 pts). Let G be a *finite* group with $|G| > 1$, such that G has no subgroup H with $\{e\} \subsetneq H \subsetneq G$. Prove that $G \cong \mathbb{Z}_p$ for some prime number p.

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