Math 100a Fall 2015 Homework 6

Due Wednesday 11/4/2015 by 5pm in HW box in basement of AP&M

Reading

All references are to Beachy and Blair, 3rd edition.

Read Sections 2.3 and Sections 3.6-3.7.

Assigned Problems (write up full solutions and hand in):

Section 2.3 #3, 5, 10, 13

Section 3.6 #1(b)(d), 12, 15(a)(c), 17, 19, 21

Problem not from the text (write up and hand in):

A. As we showed in class, the Dihedral group D_n is naturally isomorphic to a subgroup H of S_n. Namely, each motion determines a permutation by numbering the positions in the plane where the corners lie and tracking how the corners are moved around: if the motion moves the corner in position i to position j, then the corresponding permutation sends i to j.

   (a). Find explicitly the 12 permutations in H in case $n = 6$. Show how you found them.

   (b). Find the order of each element of D_6.

Optional Problem:

B. A *regular tetrahedron* T is a solid in 3-space with four sides which are equilateral triangles. It has 4 vertices and 6 edges. Consider the group G of rigid motions of T that take place in

3-space. These are the ways of rotating T in 3-space so that it occupies the same location in space afterwards. (reflections are not allowed because they would have to take place in 4-space.) Show that G is isomorphic to A_4, the alternating subgroup of S_4. (Hint: every rigid motion is determined by how the corners move around, just as we saw for dihedral groups.)