Math 100a Fall 2015 Homework 5

Due Wednesday 10/28/2015 by 5pm in HW box in basement of AP&M

Reading

All references are to Beachy and Blair, 3rd edition.

Read Section 2.3 and Section 3.6.

Assigned Problems (write up full solutions and hand in):

Section 3.4 #10, 13, 15, 24

Section 3.5 #2, 13, 16, 19, 20

Problems not from the text (also to be handed in)

A. Let m and n be positive integers. We proved the Chinese Remainder Theorem which states that if $\gcd(m, n) = 1$, then $\mathbb{Z}_{mn} \cong \mathbb{Z}_m \times \mathbb{Z}_n$.

Prove conversely that if $\gcd(m, n) \neq 1$, then $\mathbb{Z}_{mn}$ is *not* isomorphic to $\mathbb{Z}_m \times \mathbb{Z}_n$.

Optional problem (handing in not required)

B. Prove that every group of order 6 is isomorphic either to $\mathbb{Z}_6$ or else to the symmetric group S_3.

(For hints, see the sequence of problems 15-17 in Section 3.3).