Math 100a Fall 2015 Homework 1

Due Wednesday 9/30/2015 by 5pm in HW box in basement of AP&M

Reading

All references are to Beachy and Blair, 3rd edition.

Chapter 1 and Sections 2.1-2.2 should be review of Math 109 material. We will quickly review some of this material in class, but you should read it thoroughly on your own.

Begin to read Section 3.1.

Assigned Problems (write up full solutions and hand in):

Section 3.1: 2(b)(d), 10, 11, 13, 15, 22.

Problem not from the text (also to be handed in)

A. Recall that for any complex number $z = a + bi \in \mathbb{C}$ with $a, b \in \mathbb{R}$, where $i = \sqrt{-1}$, there is a well-defined complex number $e^z = e^a(\cos b + i \sin b)$. This definition of complex exponentiation satisfies the usual rule for exponents: $e^{z_1}e^{z_2} = e^{z_1+z_2}$. Also, we have $e^{2\pi ni} = 1$ for all integers n.

For any integer $n \geq 1$, define

$$G = \{e^{2\pi ki/n} | k \in \mathbb{Z}\}.$$

Show that G has precisely n distinct elements. Prove that G is a group under the operation of multiplication. It is called the *group of nth roots of unity* since the numbers in G are precisely the complex numbers z such that $z^n = 1$. Thus for each positive integer n there does exist a group with n elements.

Optional problem (handing in is not required)

B. Let G be a set with a binary operation $*$. Suppose that (i) $*$ is associative, $(ii)'$ G has an element e such that $e * a = a$ for all $a \in G$ (that is, e is a *left identity element*), and $(iii)'$ for all $a \in G$ there is an element $b \in G$ such that $b * a = e$ (that is, every element has a *left inverse*). Show that G is a group with identity element e.

(Note that you have to show that (ii) for each $a \in G$, $e * a = a = a * e$, and (iii) for each $a \in G$, there is an element $b \in G$ such that $b * a = a * b = e$).