

Math 203 - Algebraic Geometry

Instructor: Dragos Oprea, doprea@math.you-know-where.edu.

Office hours: Room 6-101, Wednesday 1:30-2:30PM.

Textbook: None. I will occasionally use Andreas Gathmann's notes available online.

Lectures: WF (11-12:20pm), 7-421.

Webpage: <http://math.ucsd.edu/~doprea/203a12.html>.

Goals: This course provides an introduction to algebraic geometry. Algebraic geometry is a central subject in modern mathematics, and an active area of research. It has connections with number theory, differential geometry, symplectic geometry, mathematical physics, string theory, representation theory, combinatorics and others.

Syllabus. We will tentatively cover the following topics:

1. (i) Affine space and affine sets. Hilbert's Nullstellensatz. The correspondence between ideals and affine sets. Zarisky topology. Irreducible affine sets. Dimension.

(ii) Functions on affine varieties. Coordinate rings. Sheaves. Morphisms. Isomorphisms. Rational and birational maps.

2. (i) Prevarieties. Gluing. Projective space, projective varieties. Examples including hypersurfaces, quadrics, Grassmannians, elliptic curves.

(ii) Homogeneous coordinate rings. Morphisms. Examples including Segre embeddings, Veronese embeddings. Rational varieties.

3. Dimension theory. Smoothness. Blowups. The 27 lines on a smooth cubic surface.

4. Affine Schemes. Schemes. Projective schemes.

5. First applications of scheme theory. Intersections in projective space, intersection multiplicities. Bezout's theorem. Hilbert functions and degree. Degree-genus formula. Applications of Bezout e.g. Pascal's mystic hexagon.

Prerequisites: Some knowledge of modern algebra at the level of Math 200 is required. Familiarity with complex analysis, basic point set topology, differentiable manifolds is helpful, but not required. Since it is hard to determine the precise background needed for this course, I will be happy to discuss prerequisites on an individual basis. If you are unsure, please don't hesitate to contact me.

Problem Sets: The final grades are based entirely on lecture attendance and weekly homework. Homework is mandatory and is a very important part of the course. The problem sets will be posted online.