Math 203A

October 10, 2022
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  - finitely generated \( k \)-algebras
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- We studied **affine varieties** and **morphisms** between them.

- Equivalence
  - affine varieties
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- We wish to define **arbitrary varieties**.
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- Equivalence
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- We wish to define arbitrary varieties.
Roadmap:

affine varieties $\mapsto$ ringed spaces
Roadmap:

affine varieties $\mapsto$ ringed spaces $\mapsto$ abstract affine varieties
Roadmap:

affine varieties $\mapsto$ ringed spaces $\mapsto$ abstract affine varieties $\mapsto$ prevarieties
Roadmap:

affine varieties $\mapsto$ ringed spaces $\mapsto$ abstract affine varieties $\mapsto$

prevarieties $\mapsto$ varieties
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$schemes$ (203B)
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schemes (203B) $\mapsto$ algebraic spaces (203 “D”) $\mapsto$ stacks (203 “D”)
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Abstract affine varieties

- We need a coordinate-free definition of affine varieties
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- This will make it easier to glue affine varieties
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**Definition**

An abstract affine variety \((X, \mathcal{O}_X)\) is a ringed space
- \(\mathcal{O}_X\) is a sheaf of \(k\)-valued functions
- \(X\) is irreducible as a topological space
- \(X\) is isomorphic to an affine variety
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Basic open sets are affine

Lemma

Let $X \subset \mathbb{A}^n$ be an affine variety. The basic open set

$$X_f = \{x \in X : f(x) \neq 0\}$$

is an abstract affine variety.
Basic open sets are affine

**Lemma**

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**Proof:**

$$Z \subset \mathbb{A}^{n+1}, \quad Z = \{(x, t) : x \in X, tf(x) - 1 = 0\}.$$
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\pi : Z &\to X_f, \quad (x, t) \mapsto x \\
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- \(X\) admits a finite cover by open sets \(X = \bigcup U_i\)

such that \((U_i, \mathcal{O}_X|_{U_i})\) is an abstract affine variety.
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Remarks

- An open set with \((U, \mathcal{O}_X|_U)\) abstract affine variety is called affine open.
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- This is similar to the definition of (complex) manifolds via charts

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- For manifolds, charts are related by (biholomorphic) transition maps

\[ g_{ij} : \phi_i(U_i \cap U_j) \to \phi_j(U_i \cap U_j). \]
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In our case, \(V_i\) are affine varieties, and the transition maps are isomorphisms of ringed spaces a.k.a. isomorphisms.
Construction of prevarieties – Gluing

▶ How do we construct prevarieties?
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- How do we construct prevarieties?

We form **prevarieties** by **gluing** two or several affine varieties.
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▶ How about gluing prevarietites?
Gluing data

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Construct the prevariety $X$: 

- **set:** $X$ is the disjoint union $X_1 \cup X_2$ modulo the equivalence $p \sim f(p)$, $p \in U_1$.

- **topology:** $X$ carries the quotient topology induced by $\sim$.

- **sheaf** $\mathcal{O}_X$: $\mathcal{O}_X(U) = \{(g_1, g_2), g_1 \in \mathcal{O}_X(U \cap X_1), g_2 \in \mathcal{O}_X(U \cap X_2) | g_1|_{U \cap U_1} = g_2|_{U \cap U_2}\}$.

- **Check:** $\mathcal{O}_X$ is a sheaf, $X$ is irreducible, every point of $X$ has an affine neighborhood.
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More gluing

**Lemma**

*Let*

- \( X_1, \ldots, X_r \) be *prevarieties*,

There is a prevariety \( X \), obtained by gluing the \( X_i \) along the morphisms \( f_{ij} \).
More gluing

Lemma

Let

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Projective line

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Affine line with double origin.

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**Affine line with double origin.**

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- $X$ can be covered by **affine opens**
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- $X$ can be covered by **affine** opens
- so $U$ can be covered by **quasi-affine** opens
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**Claim:** all $U \subset X$ open $\implies (U, O_X|_U)$ is prevariety.

- $X$ can be *covered* by affine opens
- so $U$ can be *covered* by quasi-affine opens
- each quasiaffine can be *covered* by basic open sets
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- so $U$ can be covered by **quasi-affine** opens

- each quasi-affine can be covered by **basic open sets** $=$**affine**.
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**Claim:** all $U \subset X$ open $\implies (U, O_X|_U)$ is prevariety.

- $X$ can be covered by affine opens
- so $U$ can be covered by quasi-affine opens
- each quasi affine can be covered by basic open sets $=$ affine.
- thus $U$ can be covered by affine opens too.
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- each quasi-affine can be covered by basic open sets = affine.
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**Affine and quasi-affine varieties are prevarieties.**
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**Affine and quasi-affine varieties are prevarieties.**