



Fibonacci numbers

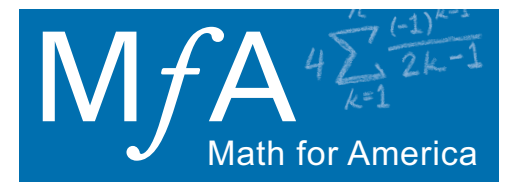
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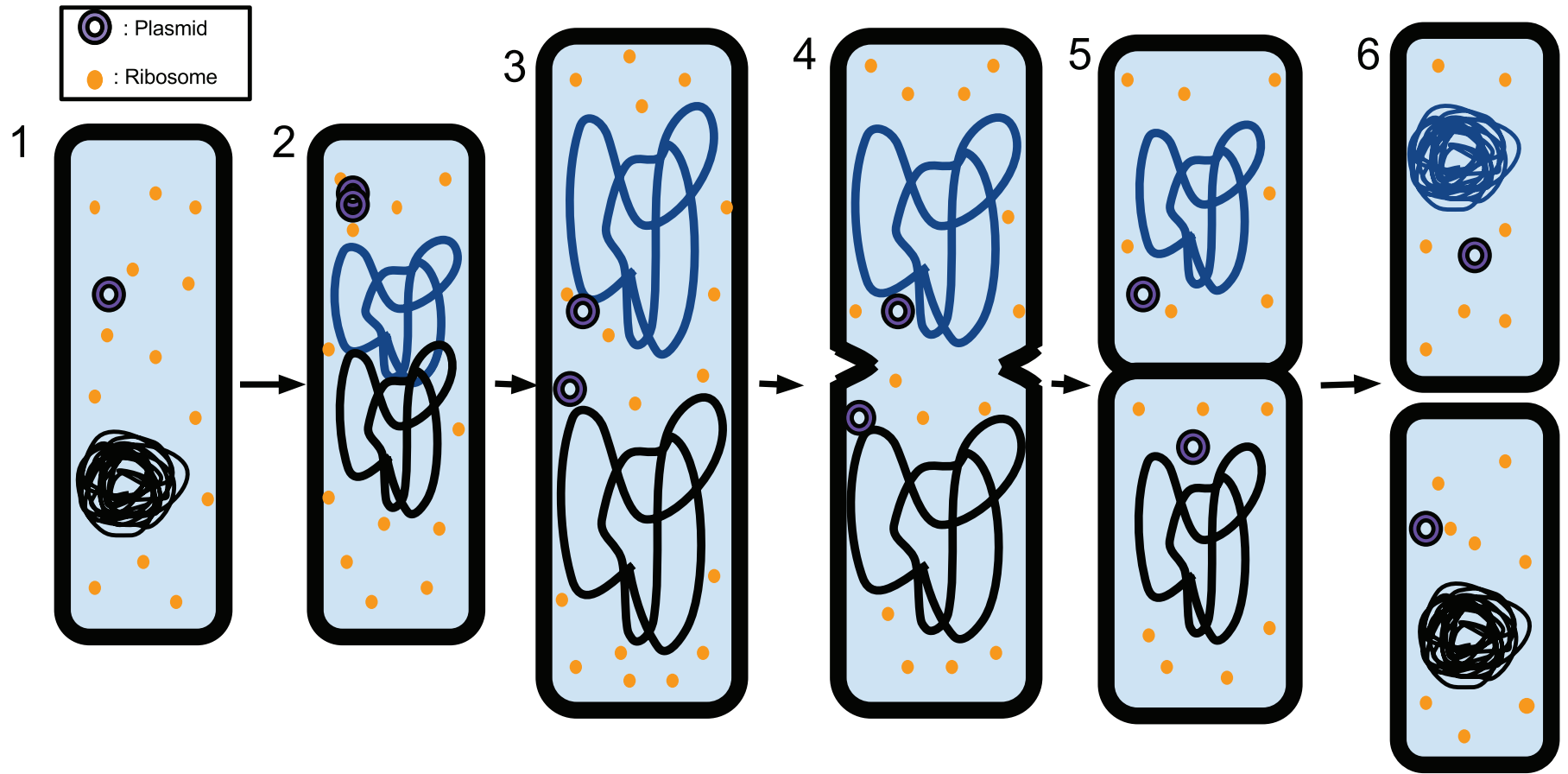
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NSF/MfA Teachers program
University of California/San Diego
La Jolla, CA, 14 July 2014

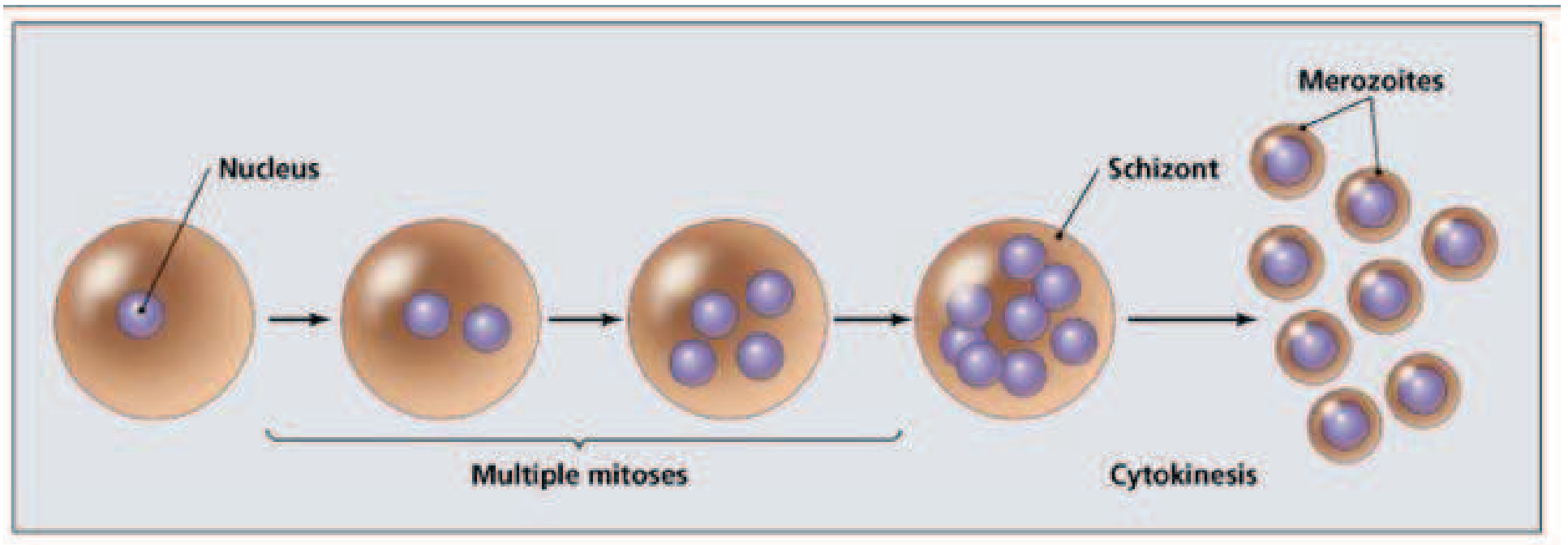


Binary fission



bacterial reproduction

Multiple fission



parasite reproduction

Leonardo of Pisa — Fibonacci (c.1170–c.1250)



Liber Abaci (1202)

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geminat. sic fit i fo mēse paru 7 er quib' i uno mēse duo pgnant
 7 geminat in teo mēse para 7 conieloz. 7 sic fit para 7 i tpo m
 se. er quib' i tpo pgnat para 7 7 fit i qto mēse para 8 er q' b'
 para 7 geminat alia para 7 quib' addit cu parit 8 fit
 ut para 7 i qto mēse. er q' b' para 7 q' geminat fuerit i tpo
 mēse si capiat i tpo mēse s. alia 8 para pgnant 7 sic fit i tero mēse
 para 7 cu q' b' addit parit 7 q' geminat i septio erit i tpo
 para 7 cu quib' addit parit 7 q' geminat i octavo mēse.
 erit i tpo para 7 cu quib' addit parit 7 q' geminat i no
 no mēse erit i tpo para 7 cu quib' addit parit 7 q' geminat i
 q' geminat i decimo. erit i tpo para 7 cu quib' addit parit
 parit 7 q' geminat i undecimo mēse. erit i tpo para 7
 cu q' b' addit parit 7 q' geminat in ultimo mēse. erit
 im. poterit unde i hio margine. quali: hoc opus sum. s. q' utrum
 pmi nūm cu fo uidet i cu 7 fm e teo. 7 teni cu q' to. 7 q' r
 tu cu q' to. 7 sic decept donec utrum decimū cu undecimo. uidet
 177 cu 777. 7 hūm' stoz canieloz sumā uidet. 777
 7 sic possit face p ordine de istant mēse mēse.

Quanis hoies fit. quoz pmi 7 sed. 7 tci hūc d' r' os. sed. itaq. 7 tci 7 q' r'
 hūc d' r' os 7 tci 7 q' r' pmi hūc d' r' os 7 tci 7 q' r' pmi 7 tci
 hūc d' r' os 7 tci 7 q' r' pmi hūc d' r' os. adde hos. ut. nūos i unū erit
 q' nūc e t' p' lū totū sume d' r' os. illoz. nū. hōimū. 7 deo q' i t' p' r'
 sumā unūq' eoz o ap' utat e q' r' d' unio t' p' p 7 reddt 7 p' eoz
 sumā. er qua si erant d' r' os pmi 7 tci 7 tci hōis. 777 remanebit
 q' r' o hōi d' r' 7 tci 7 tci si er ipit d' r' os 777 erant d' r' os 777 fi
 7 tci 7 q' r' o hōis. remanebit pmo hōi d' r' 777 Rursū si de d' r' os 777
 erant 777 .s. d' r' tci 7 q' r' o hōis. pmi hōis. remanebit fo d' r'
 er. adhuc si de d' r' os 777 erant d' r' os 777 q' r' o pmi 7 sed hōis
 remanebit teo d' r' 777 conuent itaq. d' r' os 777 pmi hōis cu
 sed. 7 cu 7 tci er cu 777 q' r' o nūm' sūa reddt 777

Tē si p' o' s' i' t' u' f' u' i' t' e' q' u' i' n' t' p' m' i' 7 f' m' hōie hūc d' r' os 777 er mē fm
 t' e' n' i' hūc d' r' os 777 er mē t' e' n' i' 7 q' r' o 777 mē q' r' o 7 p' m' i' 777
 s' i' m' i' l' e' t' h' p' o' s' i' t' u' s' q' u' i' s' o' l' u' i' p' o' s' s' e' q' u' i' s' n' . S' u' n' t' u' t' i' p' e' q' s' o' l' u' i' p' o' s' s' e'
 ab hys qui solui n' possit cognoscit. tale e' t' m' d' i' m' e' u' i' d' e' t' u' . u' i' d' e' t' e' z
 ut addit nūm. pmi 7 fi. cu nūo tci 7 q' r' o. 7 si eoz sumā equal' fuit
 nūo fi. tci 7 q' r' o 7 pmi. tē solubil' erit q' s' t' o. si at' sequal' fuit. tē eā
 nō possit solui cognoscit ut i huc q' s' t' i' o' n' e' i' q' p' m' i' sed' i' s' t' 777 er
 tē 7 q' r' o hūc 777 q' mē om' n' i' s' hūc d' r' os 777 .s. si sed' 7 tci

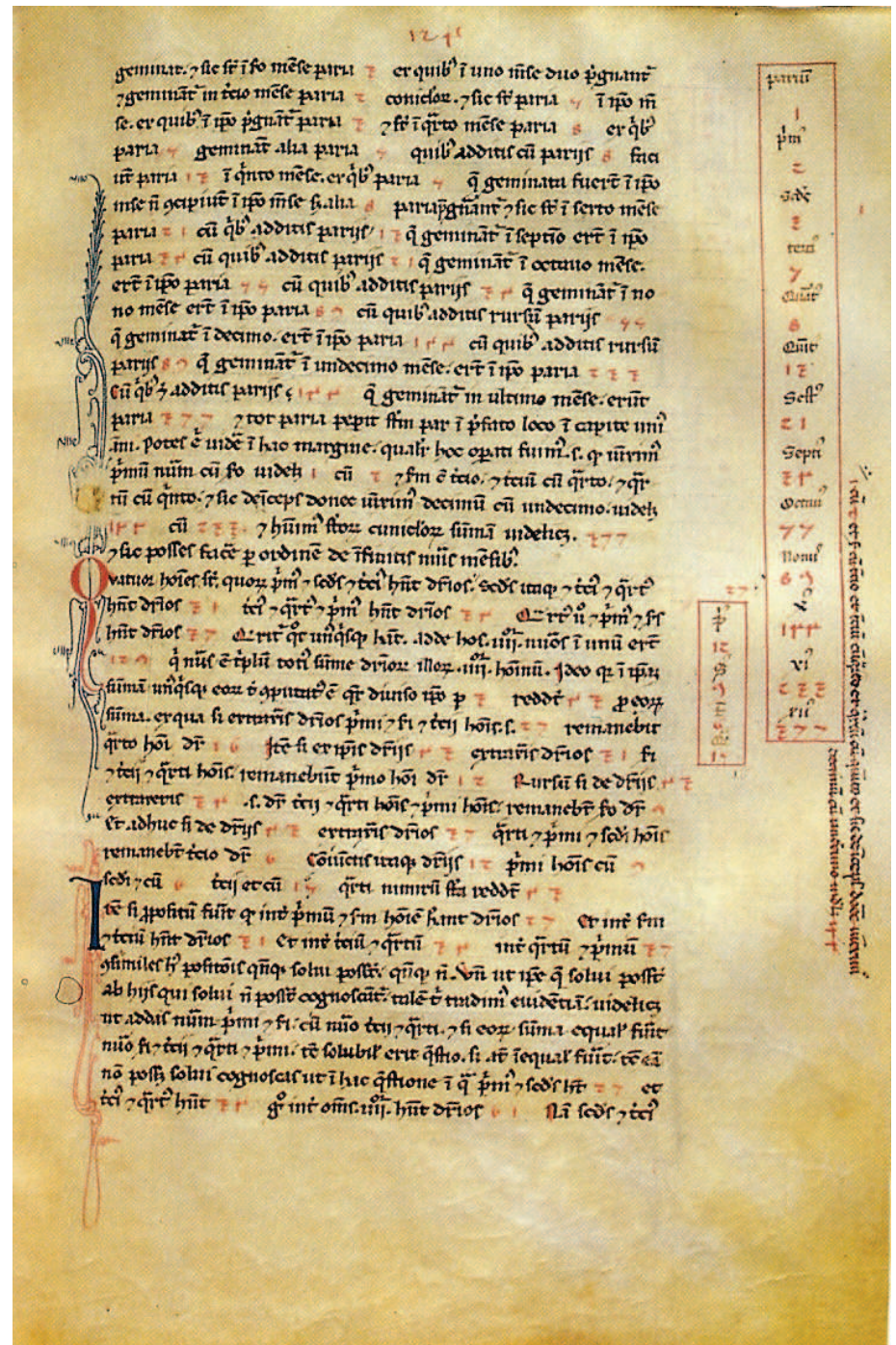
paru
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1. et er f' a' t' i' o' er t' u' t' a' q' u' o' er f' i' c' e' d' e' c' e' p' t' . A' d' d' e' u' i' d' e' t' u' t' .
 d' e' c' i' m' i' u' s' a' d' u' n' d' e' c' i' m' o' u' i' d' e' t' . 777

Liber Abaci (1202)

Quot paria coniculorum in uno anno ex uno pario germinentur.

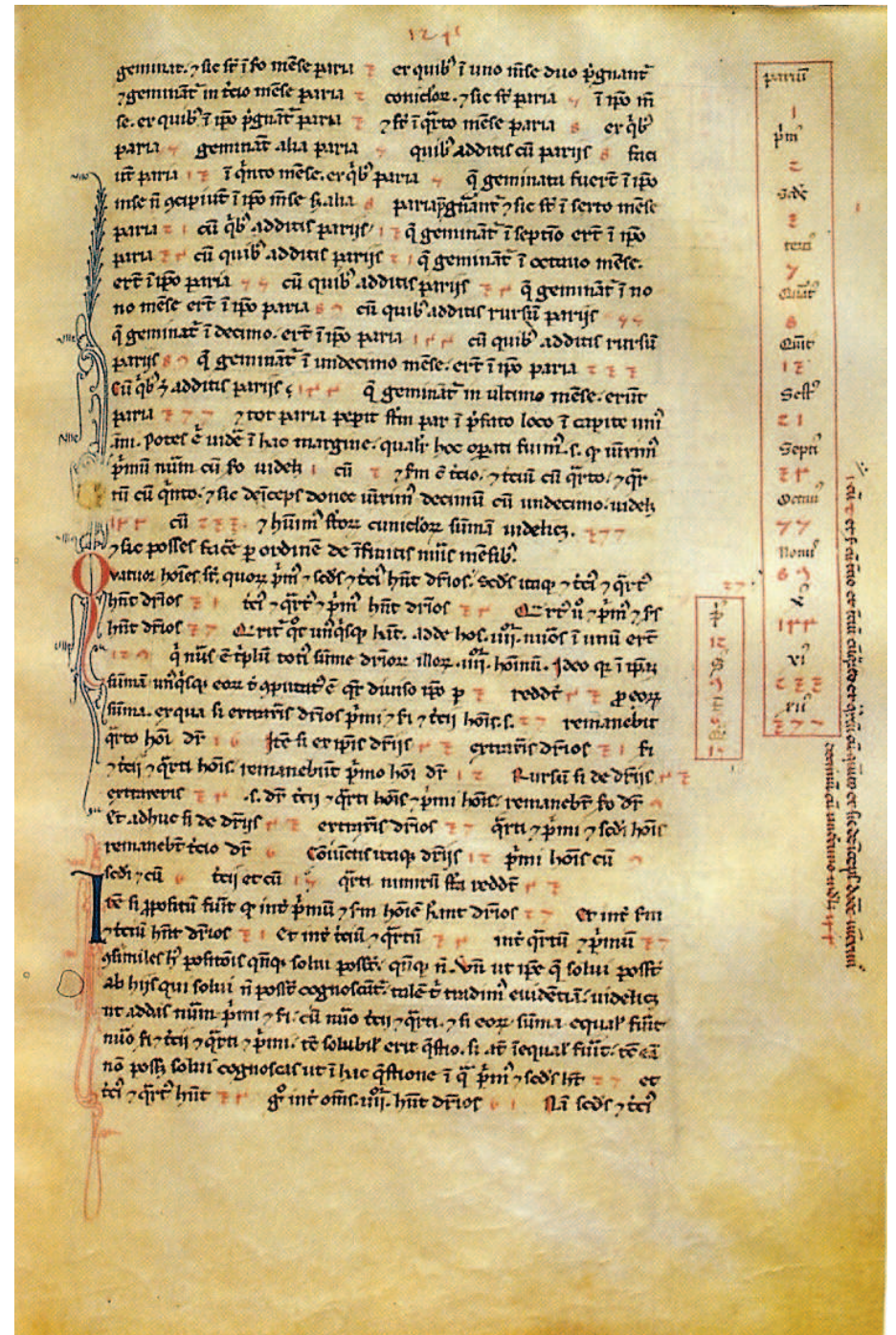
Quidam posuit unum par cuniculorum in quodam loco, qui erat undique pariete circumdatus, ut sciret, quot ex eo paria germinerentur in uno anno, cum natura eorum sit per singulum mensem aliud par germinare, et in secundo mense ab eorum natiuitate germinant.



Liber Abaci (1202)

How many pairs of rabbits are created by one pair in one year?

A man placed one pair of rabbits together in an enclosure, so he might know how many are created from a pair in one year, when it is the nature of them in a single month to bear another pair, and in the second month those born to bear also.



Phyllotaxis



pinecone

Phyllotaxis



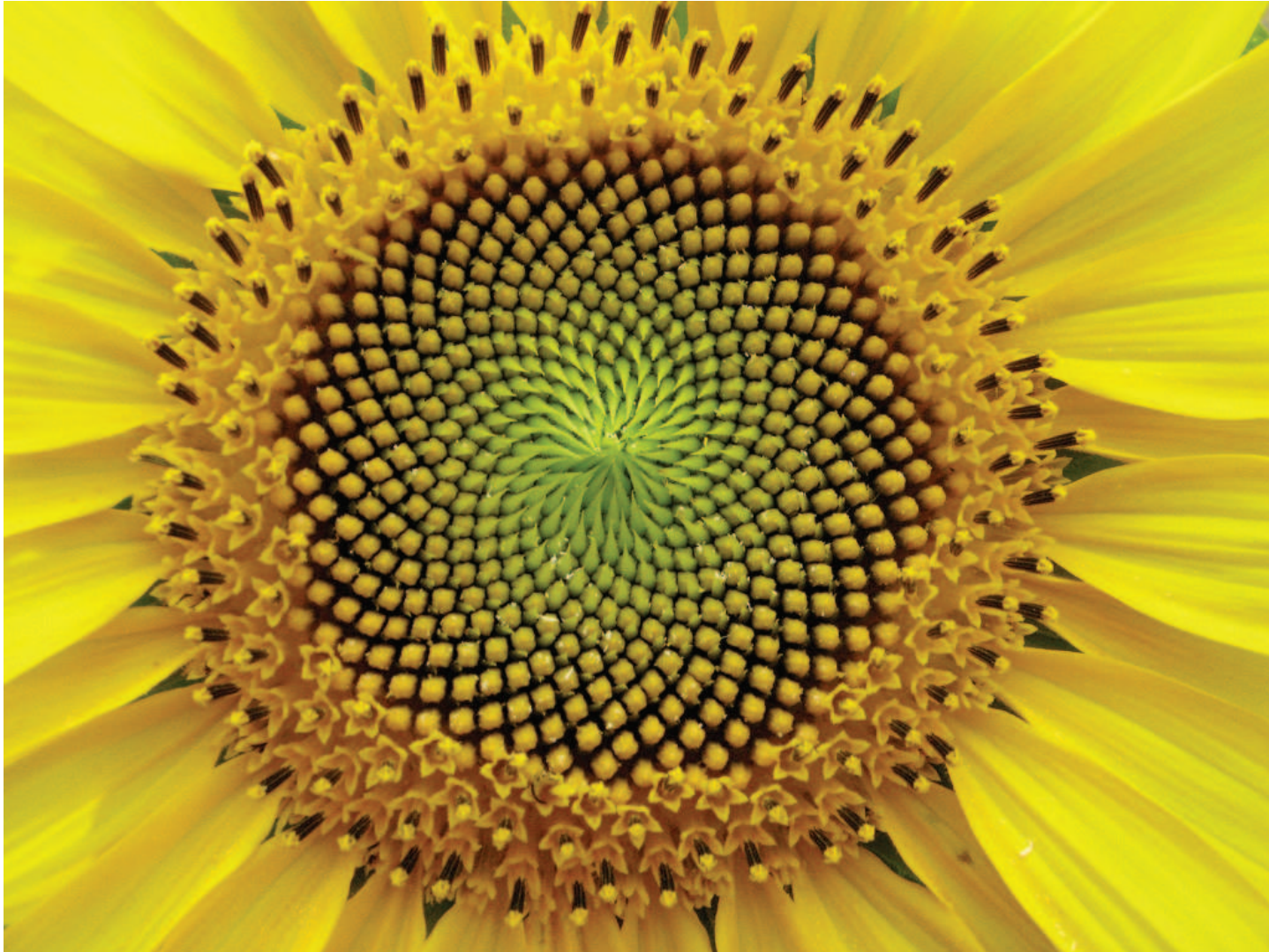
8 spirals counterclockwise

Phyllotaxis



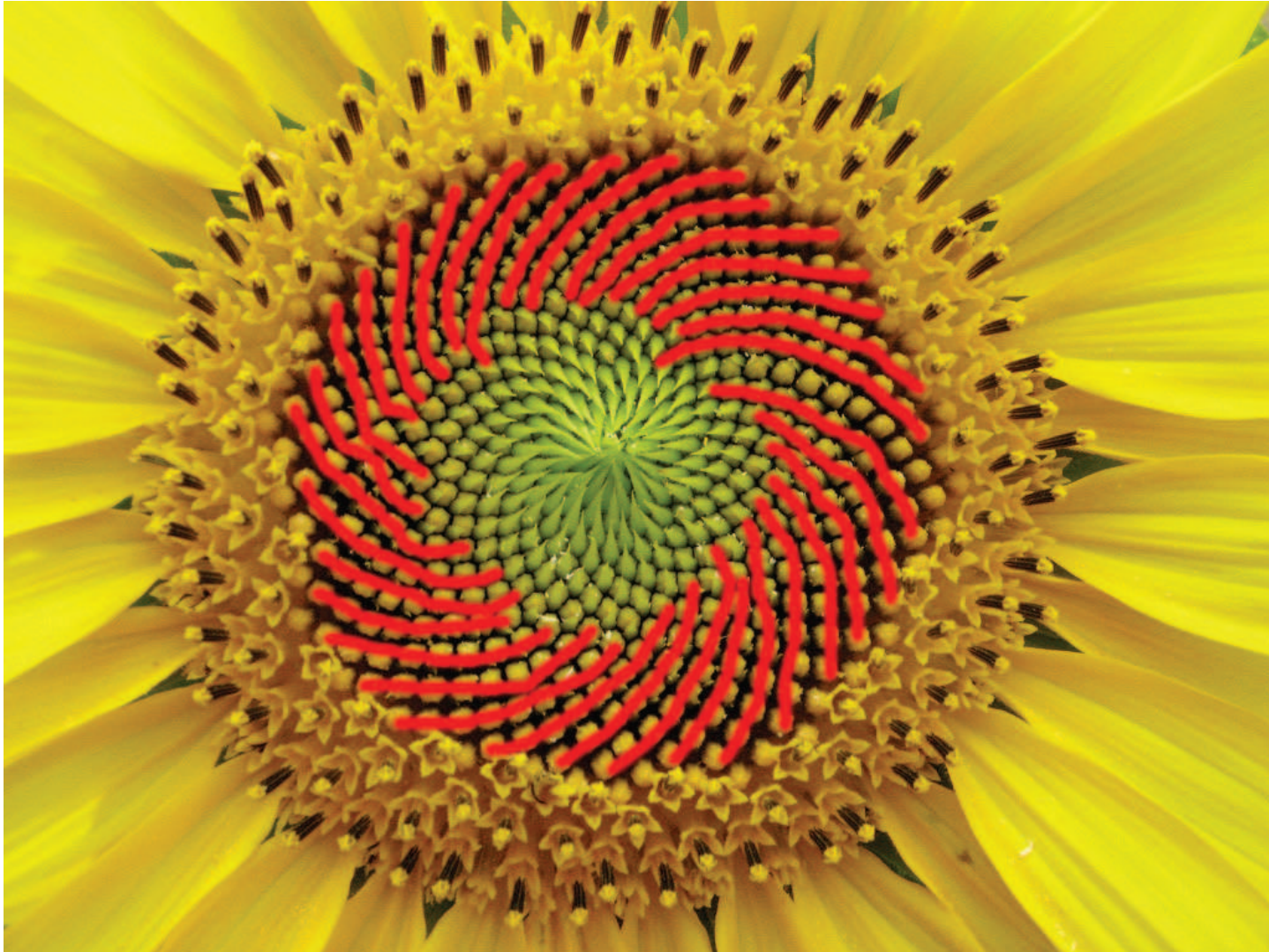
13 spirals clockwise

Phyllotaxis



sunflower

Phyllotaxis



34 spirals clockwise

Phyllotaxis

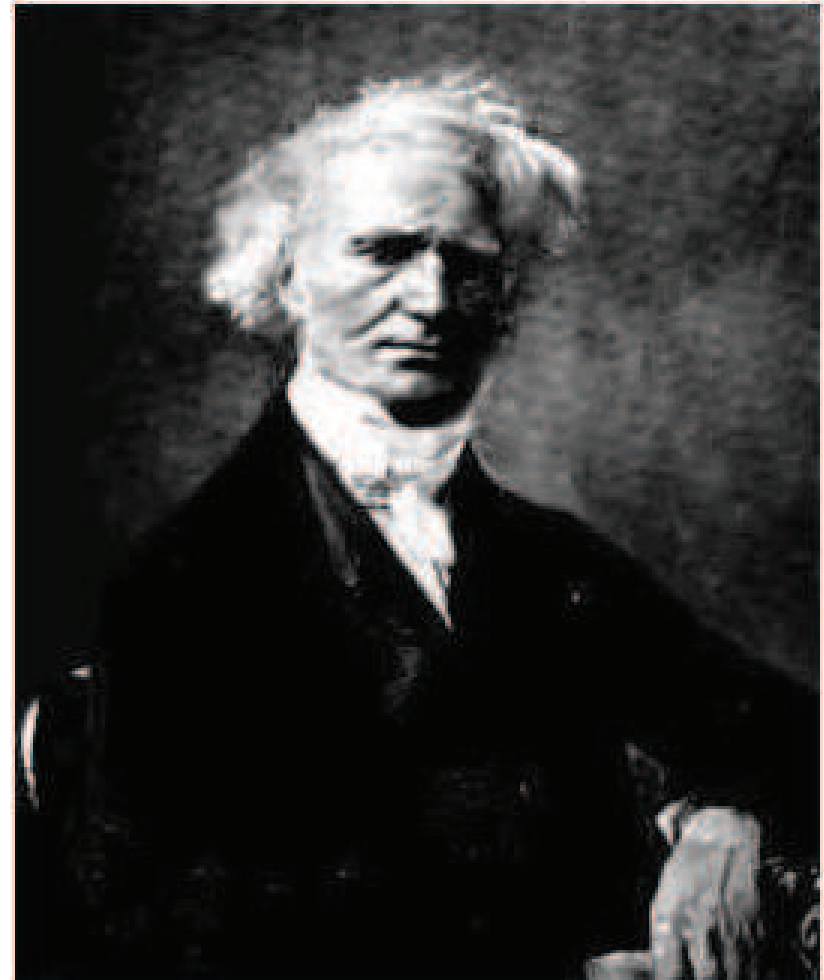


55 spirals counterclockwise

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right]$$



Abraham de Moivre
1667–1754



Jacques Philippe Marie Binet
1786–1856

Questions

1. What is the parity (evenness/oddness) of F_n ?
2. Use the recurrence relation to define F_n for negative integers n .
3. Find a formula for finite Fibonacci series: $F_1 + F_2 + \cdots + F_n$.
4. Find a formula for: $F_1 + F_3 + \cdots + F_{2n-1}$.
5. Find a formula for: $F_1^2 + F_2^2 + \cdots + F_n^2$.
6. Use the fact that $\phi^2 = \phi + 1$ to find a formula for ϕ^n .
7. Find a formula for F_n involving ϕ , but not $1/\phi$.
8. What is $\lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n}$?

Phyllotaxis



calla lily: 1 petal

Phyllotaxis



euphorbia: 2 petals

Phyllotaxis



trillium: 3 petals

Phyllotaxis



buttercup: 5 petals

Phyllotaxis



columbine: 5 petals

Phyllotaxis



delphinium: 5 petals

Phyllotaxis



bloodroot: 8 petals

Phyllotaxis



delphinium: 8 petals

Phyllotaxis



signet marigold: 8 petals

Phyllotaxis



black-eyed Susan: 13 petals

Phyllotaxis



13 petals

Phyllotaxis



chrysanthemum carinatum: 21 petals

Phyllotaxis



daisy: 21 petals

Phyllotaxis

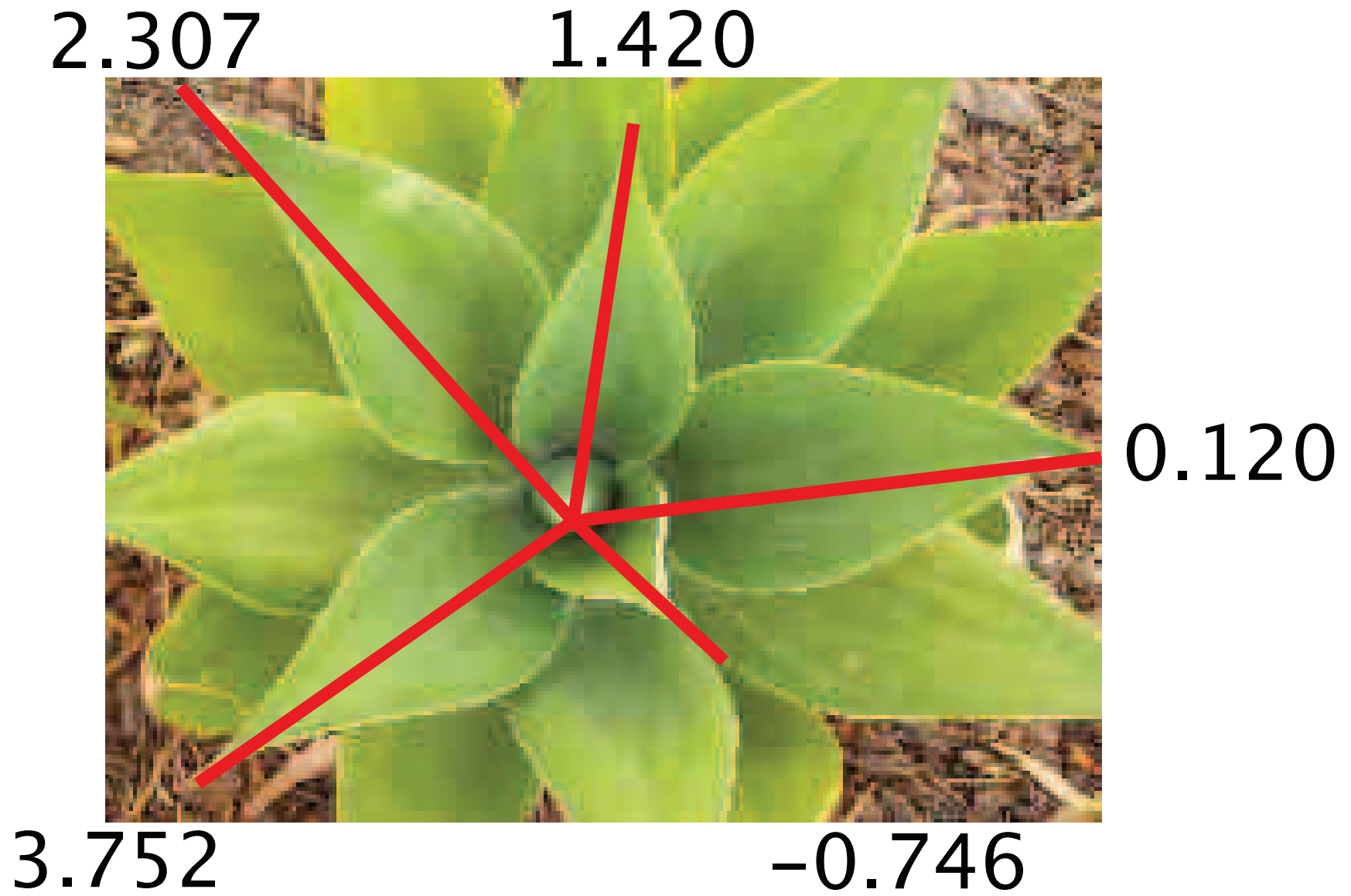


sunflower: 21 petals

Phyllotaxis



Phyllotaxis



Questions

1. What are the first several convergents of $\psi = 1 + \sqrt{2}$?
2. Find a quadratic equation satisfied by ψ (and $-1/\psi$).
3. What is the corresponding recurrence relation?
4. What are the numbers analogous to the Fibonacci numbers?

And some geometry questions ...