MATH 109 Midterm 1 17 October 2003

This is the first midterm from the last time I taught this class. You may find it a bit difficult, but it will be good practice to try to do all the problems.

1. a. How many ways are there to assign truth values to the variables A, B, C so that the propositional expression

$$(A \vee \neg B) \wedge (\neg A \vee C) \wedge (B \vee \neg C)$$

is true?

b. In a complicated propositional expression like the one in (a), a part that is enclosed in parentheses is called a *clause*. Clauses that contain only $\neg$s and $\vee$s (and variables) are called *disjunctive clauses*. A propositional expression, like the one in (a), that is the conjunction (and) of disjunctive clauses, each with a single $\vee$, is said to be in *2-conjunctive normal form*, abbreviated 2-CNF. What is the shortest 2-CNF propositional expression that has no truth assignment for the variables A, B, C that makes it true?

2. Let x and y be real numbers such that $x < y$. Prove the following statement: If x and y are rational numbers then there are infinitely many rational numbers r such that $x < r < y$.

3. Let $Q(x, y)$ be the propositional function "x and y are rational numbers", and let $I(x, y)$ be the propositional function "there are infinitely many rational numbers r such that $x < r < y$".

a. Write the statement in problem 2 in terms of these propositional functions, using quantifiers.

b. What is the negation of the statement in problem 2?

c. Let $E(x, y)$ be the propositional function "every real number r such that $x < r < y$, is rational". Is $E(x, y)$ equivalent to $I(x, y)$? Why or why not?

4. Let A and B be sets. Let $D = (A \cup B) \setminus (A \cap B)$. Prove that $|D| = 0$ if and only if $A = B$.

Extra Credit: Can each point in the plane be colored with one of three colors in such a way that *every* equilateral triangle with sides of length 1 has one vertex of each color? Prove that your answer is true.