Name: Solutions PID: ____________

NOTE: You must show the steps necessary to arrive at your answer unless otherwise noted. Use your judgment, if you can't do the entire problem in your head, then you probably should write down at least some intermediate steps.

This assignment has 9 pages. There are 61 total points.

Problem 1 (4 points). Compute the following, or state if they are undefined:

(a) $\sec\left(\frac{\pi}{2}\right)$

$\sec(\frac{\pi}{2}) = \frac{1}{\cos(\frac{\pi}{2})} = \frac{1}{0} \longrightarrow \sec(x)$ is undefined at $x = \frac{\pi}{2}$

(b) $\tan\left(\frac{4\pi}{3}\right)$

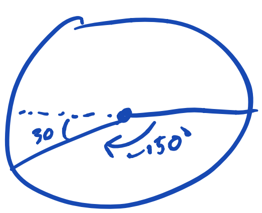
$$\tan\left(\frac{4\pi}{3}\right) = \frac{\sin(\frac{4\pi}{3})}{\cos(\frac{4\pi}{3})} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} \cdot \frac{2}{2} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

(c) $\tan(0°)$

$$\tan(0°) = \frac{\sin(0°)}{\cos(0°)} = \frac{0}{1} = 0$$

(d) $\csc(-150°)$

$$\csc(-150°) = \frac{1}{\sin(-150°)} = \frac{1}{-\frac{1}{2}} = -2$$



Problem 2 (8 points). Suppose $\sin(\theta) = 0.4$ and $\cos(\theta) = .92$. Compute the following to two decimal places. You *may* use a calculator - *you must write down the calculation you use to find the answer*.

(a) $cot(\theta)$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{.92}{.4} = 2.30$$

(b) $csc(\theta)$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{.4} = 2.50$$

(c) $tan(\theta)$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{.4}{.92} = 0.43$$

(d) $sec(\theta)$

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{.92} = 1.09$$

Problem 3 (4 points). What is the domain and range of $f(x) = \sin(x)$? What is the domain and range of $g(x) = \cos(x)$?

Domain of sin(x) is all real numbers.
Range of sin(x) is $-1 \le y \le 1$.

Domain of cos(x) is all real numbers.
Range of cos(x) is $-1 \le y \le 1$.

Problem 4 (8 points). Find the **range** of the following functions:

(a) $r(y) = 4\sin(y)$

↑ Vertical stretch by factor of 4.

Range: $[-4, 4]$

(b) $p(a) = -3\cos(a)$

↑ Vertical stretch by factor of 3 (and vertical reflection)

Stretch changes range from $[-1, 1]$ to $[-3, 3]$.
V. Reflection leaves range unchanged because $[-3, 3]$ is symmetric vertically

(c) $k(t) = \cos(t) + 2$

↑ vertical shift up by 2,

changes range from $[-1, 1]$ to $[1, 3]$ (+2)

(d) $g(s) = -2\sin(t) + 5$

↑ Vertical stretch by 2 and vertical reflection

vertical shift up by 5

Order of transformations rule: Stretches/Reflections before shifts (see Chap. 1)

Range of $\sin(t)$: $[-1, 1]$
↓ ×2
Range of $2\sin(t)$: $[-2, 2]$
↓ ×(−1)
Range of $-2\sin(t)$: $[-2, 2]$
↓ +4
Range of $-2\sin(t)+5$: $[3, 7]$

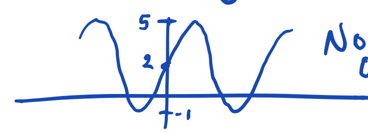
Problem 5 (3 points). True or False.

True The period of $f(\alpha) = 2\cos(\alpha)$ is 2π.

False $g(x) = 3\sin(x) + 2$ is an odd function.

True $h(z) = -2\cos(z)$ is an even function.

$g(-x) = 3\sin(-x) + 2$
$= -3\sin(x) + 2$
$\neq -g(x)$



Not odd

Problem 6 (4 points). If $\cos(\beta) = -0.43$, what is $\cos(-\beta) + \cos(\beta + 4\pi)$? *Hint: Use your knowledge of even/odd functions and periodic functions*

Since $\cos(x)$ is an even function,
$\cos(-\beta) = \cos(\beta) = -0.43$

Since $\cos(x)$ is periodic with period 2π,
$\cos(x+4\pi) = \cos(x+2\pi)$ and
$\cos(x+2\pi) = \cos(x)$.

So $\cos(\beta + 4\pi) = \cos(\beta) = -0.43$

Thus $\cos(-\beta) + \cos(\beta+4\pi) = -0.43 + (-0.43)$
$= \underline{-0.86}$

Problem 7 (5 points). Find the period of the following sinusoidal functions:

(a) $T(x) = 3\sin(x) + 2$ ← vertical shift up by 2 units

↑ Vertical stretch by factor of 3

No horizontal stretch → period is $\boxed{2\pi}$

(b) $k(y) = \sin(2y)$

↑ Horizontal stretch by factor of $\frac{1}{2}$

New period $= \frac{1}{2} \cdot 2\pi = \boxed{\pi}$

(c) $r(s) = \sin(-2s)$

↑ Horizontal stretch by factor of 2, with horizontal reflection.

New period $= \frac{1}{2} \cdot 2\pi = \boxed{\pi}$

(d) $q(u) = 2\cos(3u)$

v stretch by factor of 2

↑ H. stretch by factor of $\frac{1}{3}$

New period $= \frac{1}{3} \cdot 2\pi = \boxed{\frac{2\pi}{3}}$

(e) $w(x) = -4\cos\left(\frac{x}{10}\right)$

vertical stretch by factor of 4 and vertical reflection

Horizontal stretch by factor of $\frac{1}{1/10} = 10$.

New period $= 10 \cdot 2\pi = \boxed{20\pi}$

Problem 8 (8 points). Compute the following without the use of a calculator.

(a) sin(600)

$-600°$ is the same as $120°$ $(-600+360+360=120)$

$\sin(120°)=\frac{\sqrt{3}}{2} \longrightarrow$ $\boxed{\sin(-600°)=\frac{\sqrt{3}}{2}}$

(b) tan(3⇡)

3π is the same angle as π $(3\pi-2\pi=\pi)$

$\tan(\pi)=\frac{\sin(\pi)}{\cos(\pi)}=\frac{0}{1}=0 \longrightarrow$ $\boxed{\tan(3\pi)=0}$

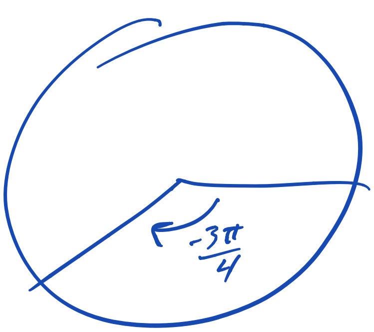
(c) cos ⇣ $\frac{25⇡}{6}$ ⌘

$\frac{25\pi}{6}$ is the same angle as $\frac{\pi}{6}$ $\left(\frac{25\pi}{6}-2\pi-2\pi=\frac{\pi}{6}\right)$

$\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$, so $\boxed{\cos\left(\frac{25\pi}{6}\right)=\frac{\sqrt{3}}{2}}$

(d) sec ⇣ $\frac{3⇡}{4}$ ⌘

$\sec\left(-\frac{3\pi}{4}\right)=\frac{1}{\cos\left(-\frac{3\pi}{4}\right)}=\frac{1}{-\frac{\sqrt{2}}{2}}=\boxed{\frac{-2}{\sqrt{2}}}$



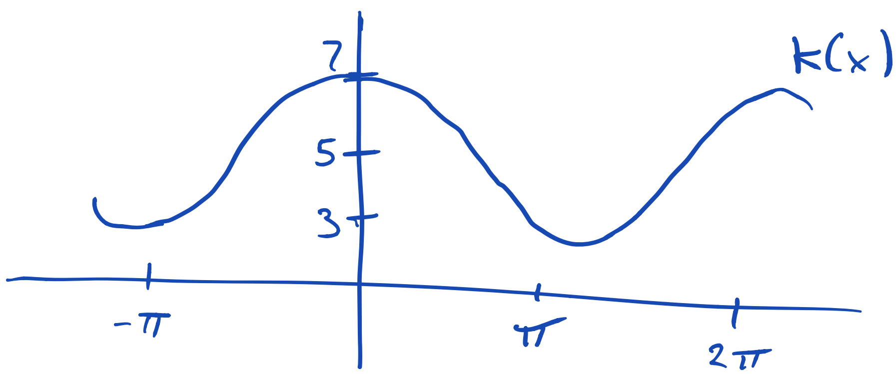
Optional simplification:

$\frac{-2}{\sqrt{2}}=\frac{-2\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}=\frac{-2\sqrt{2}}{2}=-\sqrt{2}$

Problem 9 (4 points). Describe the function $k(x) = 2\cos(x) + 5$ in terms of transformations of $f(x) = \cos(x)$.

k(x) is f(x) vertically stretched by a factor of 2 and vertically shifted up by 5 units.

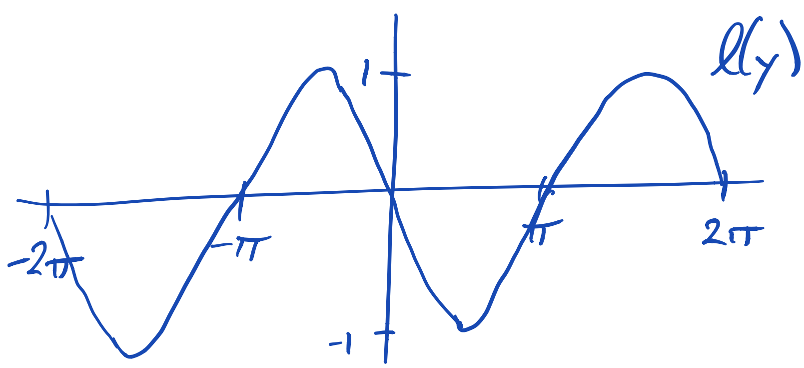
Sketch a graph of $k(x)$ without the use of a calculator. Label the tick marks on your axes so the units are clear.



Problem 10 (4 points). Describe the function $l(y) = \sin(-\pi + y)$ in terms of transformations of $g(y) = \sin(y)$.

l(y) is g(y) shifted right by π units

Sketch a graph of $l(y)$ without the use of a calculator. Label the tick marks on your axes so the units are clear.



Problem 11 (9 points). Simplify the following expressions into a single trig function with no fractions.
These are a bit more difficult than the problems we did in lecture. If you do not know where to start, try a few different methods (the solution isn't always clear). As always, be sure to go back and check your work at the end too.

(a) $\frac{1-\sin^2(\theta)}{\sin^2(\theta)}$ *(Hint: The Pythagorean Identity says that* $\cos^2(\theta)+\sin^2(\theta)=1$*)*

$1-\sin^2(\theta)=\cos^2(\theta)$, so

$$\frac{1-\sin^2(\theta)}{\sin^2(\theta)}=\frac{\cos^2(\theta)}{\sin^2(\theta)}=\boxed{\cot^2(\theta)}$$

(b) $\frac{\cot(x)}{\csc(x)}\cdot\sec(x)$

$$\frac{\cot(x)}{\csc(x)}\cdot\sec(x)=\frac{\frac{\cos(x)}{\sin(x)}}{\frac{1}{\sin(x)}}\cdot\frac{1}{\cos(x)}=\frac{\frac{\cos(x)}{\sin(x)}\cdot\frac{1}{\cos(x)}}{\frac{1}{\sin(x)}}$$

$$=\frac{\frac{1}{\sin(x)}}{\frac{1}{\sin(x)}}=\boxed{1}$$

(c) $\frac{\tan(y)}{\sec(y)-\cos(y)}$

$$\frac{\frac{\sin(y)}{\cos(y)}}{\frac{1}{\cos(y)}-\cos(y)}\cdot\frac{\cos(y)}{\cos(y)}=\frac{\sin(y)}{1-\cos^2(y)}=\frac{\sin(y)}{\sin^2(y)}=\frac{1}{\sin(y)}=\boxed{\csc(y)}$$

[OPTIONAL]
Survey Questions.
How much time during the week did you spend working on this assignment specifically?

What are the top two resources you use to study for the midterms? (the word "resource" can mean whatever makes sense to you)