Name: Solutions PID: ____________

NOTE: You must show the steps necessary to arrive at your answer unless otherwise noted. Use your judgment, if you can't do the entire problem in your head, then you probably should write down at least some intermediate steps.

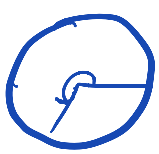
This assignment has 7 pages. There are 48 total points.

Problem 1 (3 points). Let $\theta = 240°$.

(a) What quadrant is θ in?

3rd Quadrant

(b) What is the reference angle of θ?



60°

(c) What are $\sin(\theta)$ and $\cos(\theta)$?

$\sin(240°) = -\sin(60°) = \frac{-\sqrt{3}}{2}$

$\cos(240°) = -\cos(60°) = \frac{-1}{2}$

Problem 2 (3 points). Let $\theta = -210°$.

(a) What quadrant is θ in?

2nd Quadrant

(b) What is the reference angle of θ?

30°

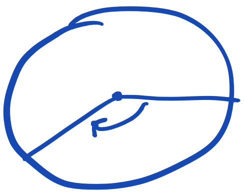
(c) What are $\sin(\theta)$ and $\cos(\theta)$?

$\sin(-210^\circ) = \sin(30^\circ) = \frac{1}{2}$

$\cos(-210^\circ) = -\cos(30^\circ) = -\frac{\sqrt{3}}{2}$

Problem 3 (3 points). Let $\theta = -\frac{3\pi}{4}$.

(a) What quadrant is θ in?



3rd Quadrant

(b) What is the reference angle of θ **(in radians)**?

$\frac{\pi}{4}$

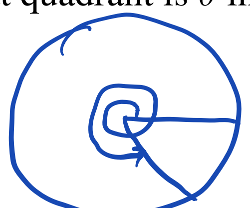
(c) What are $\sin(\theta)$ and $\cos(\theta)$?

$\sin\left(-\frac{3\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\cos\left(-\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

Problem 4 (3 points). Let $\theta = \frac{15\pi}{4}$.

(a) What quadrant is θ in?



4th Quadrant

(b) What is the reference angle of θ **(in radians)**?

$\frac{\pi}{4}$

(c) What are $\sin(\theta)$ and $\cos(\theta)$?

$\sin\left(\frac{15\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\cos\left(\frac{15\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Problem 5 (4 points). True or False.

False ______ Let θ be an angle such that $\sin(\theta) > 0$. It is possible that θ is in the third quadrant.

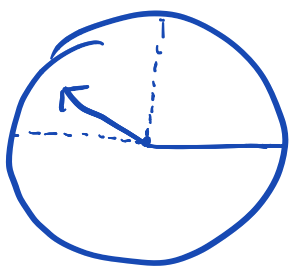
True ______ Let γ be an angle such that $\sin(\gamma) < 0$. It is possible that γ is in the fourth quadrant.

False ______ Let β be an angle such that $\cos(\beta) < 0$. It is possible that β is in the fourth quadrant.

True ______ Let α be an angle such that $\cos(\alpha) < 0$ and $\sin(\alpha) > 0$. It is possible that α is in the second quadrant.

Problem 6 (4 points). Let α be an angle such that $\sin(\alpha) = \frac{3}{5}$, and α is in the second quadrant. What is $\cos(\alpha)$? *(HINT: recall that* $\cos(\theta)^2 + \sin(\theta)^2 = 1$ *for any angle* θ*)*

2nd Quadrant



Pythagorean Identity:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$\Rightarrow \cos^2(\theta) + \left(\frac{3}{5}\right)^2 = 1$$

$$\Rightarrow \cos^2(\theta) + \frac{9}{25} = \frac{25}{25}$$

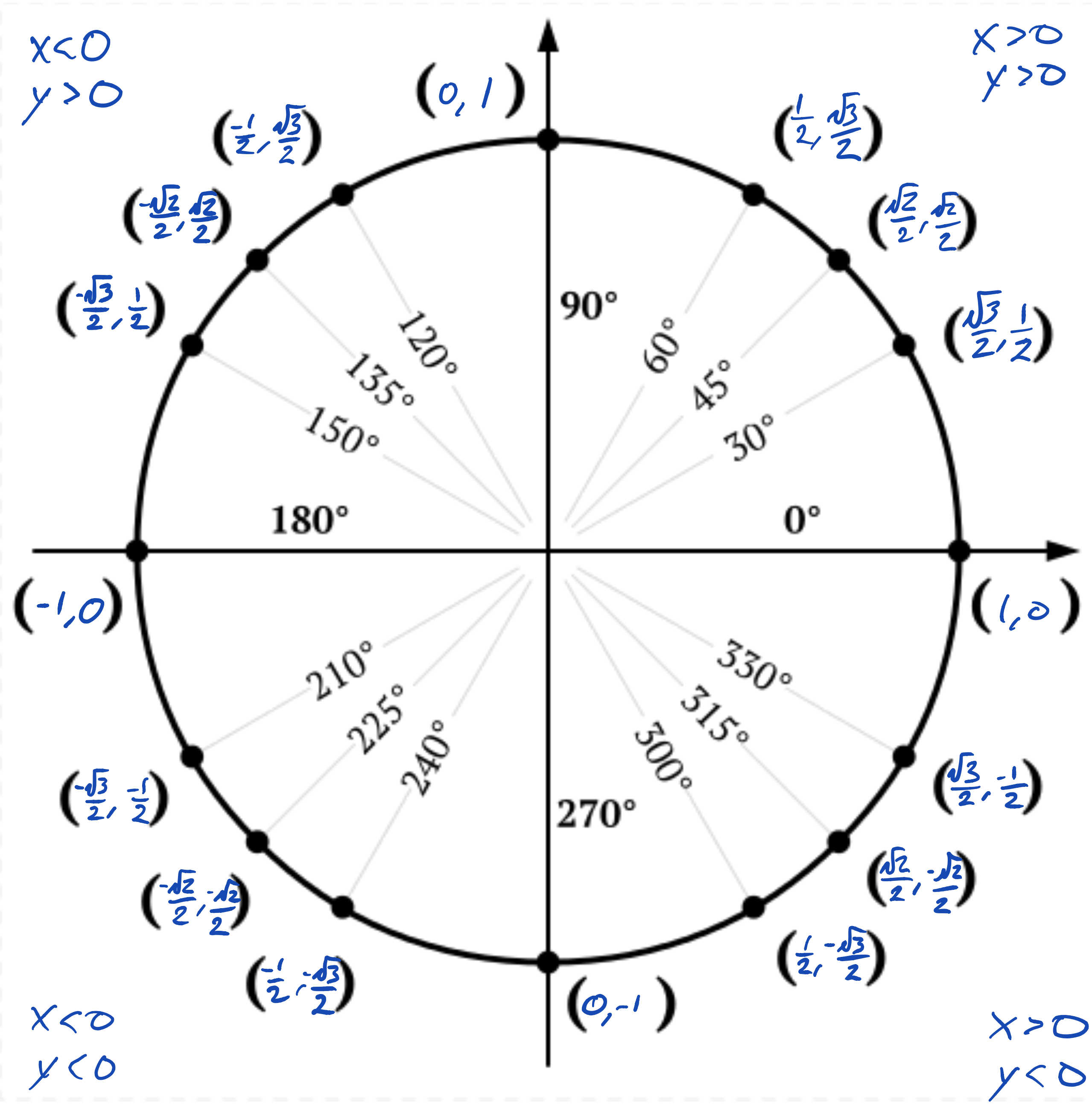
$$\Rightarrow \cos^2(\theta) = \frac{16}{25}$$

$$\Rightarrow \cos(\theta) = \pm\sqrt{\frac{16}{25}}$$

$$\Rightarrow \cos(\theta) = \pm\frac{4}{5}$$

BUT θ is in the second quadrant, so $\cos(\theta) < 0$. Thus $\boxed{\cos(\theta) = -\frac{4}{5}}$

Problem 7 (6 points). Fill in the coordinate of each point on the unit circle below.
Note: You will be required to do this with no notes on quizzes/exams in the future!



Problem 8 (2 points). Find the coordinates of the point on a circle of radius 6 at an angle of $\frac{\pi}{3}$ radians

$$x = r \cdot \cos(\theta) = 6 \cdot \cos\left(\tfrac{\pi}{3}\right) = 6 \cdot \tfrac{1}{2} = 3$$

$$y = r \cdot \sin(\theta) = 6 \cdot \sin\left(\tfrac{\pi}{3}\right) = 6 \cdot \tfrac{\sqrt{3}}{2} = 3\sqrt{3}$$

$$(x, y) = (3, 3\sqrt{3})$$

Problem 9 (4 points). The point $(-7, -7\sqrt{3})$ lies on the circle of radius 14. At what angle around the circles does this point lie?

$$x = -7 = r \cdot \cos(\theta) = 14 \cdot \cos(\theta)$$

$$\Rightarrow -7 = 14\cos(\theta)$$

$$\Rightarrow \cos(\theta) = -\tfrac{1}{2}$$

$$y = -7\sqrt{3} = r \cdot \sin(\theta) = 14\sin(\theta)$$

$$\Rightarrow -7\sqrt{3} = 14\sin(\theta)$$

$$\Rightarrow \sin(\theta) = \frac{-\sqrt{3}}{2}$$

What angle satisfies both $\cos(\theta) = \frac{-1}{2}$ and $\sin(\theta) = \frac{-\sqrt{3}}{2}$?

$\theta = 120^\circ$ or $\underline{240^\circ}$

$\theta = \underline{240^\circ}$ or 300°

$$\underline{\theta = 240^\circ}$$

Problem 10 (8 points). Compute the following:

(a) $\tan(135°)$

$$\tan(135°) = \frac{\sin(135°)}{\cos(135°)} = \frac{\frac{\sqrt{2}}{2}}{\frac{-\sqrt{2}}{2}} = -1$$

(b) $\csc(\frac{\pi}{6})$

$$\csc\left(\frac{\pi}{6}\right) = \frac{1}{\sin(\frac{\pi}{6})} = \frac{1}{1/2} = 2$$

(c) $\cot(\frac{4\pi}{3})$

$$\cot\left(\frac{4\pi}{3}\right) = \frac{\cos(\frac{4\pi}{3})}{\sin(\frac{4\pi}{3})} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{-1}{2} \cdot \frac{2}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

(d) $\sec(180°)$

$$\sec(180°) = \frac{1}{\cos(180°)} = \frac{1}{-1} = -1$$

Problem 11 (8 points). Simplify the following expressions into a single trig function with no fractions:

(a) $\cot(\theta)\sin(\theta)$

$$\cot(\theta)\sin(\theta) = \frac{\cos(\theta)}{\sin(\theta)} \cdot \sin(\theta) = \cos(\theta)$$

(b) $\sec(\theta)\csc(\theta)$

Not graded

In Week 10, we learn that $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

Using this:

$$\sec(\theta)\csc(\theta) = \frac{1}{\cos(\theta)} \cdot \frac{1}{\sin(\theta)} = \frac{1}{\sin(\theta)\cos(\theta)}$$

$$= \frac{1}{\frac{1}{2}\sin(2\theta)} = 2 \cdot \frac{1}{\sin(2\theta)} = 2\csc(\theta)$$

(c) $\csc(\theta)\cos(\theta)$

$$\csc(\theta)\cos(\theta) = \frac{1}{\sin(\theta)} \cdot \cos(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \cot(\theta)$$

(d) $\frac{1-\sin(\theta)^2}{\sin(\theta)^2}$

Since $\sin^2(\theta) + \cos^2(\theta) = 1$, it's true that $\cos^2(\theta) = 1 - \sin^2(\theta)$, so

$$\frac{1-\sin^2(\theta)}{\sin^2(\theta)} = \frac{\cos^2(\theta)}{\sin^2(\theta)} = \cot^2(\theta).$$

[OPTIONAL]

Survey Questions.

1. Do you find the lectures to go:

too fast **too slow** **roughly the right speed**

2. How do you feel about Midterm 2?

Not confident **Somewhat not confident** **No Idea** **Somewhat confident** **Confident**