Name: Solutions PID: ____________

NOTE: You must show the steps necessary to arrive at your answer unless otherwise noted. Use your judgment, if you can't do the entire problem in your head, then you probably should write down at least some intermediate steps.

This assignment has 9 pages. There are 42 total points.

Problem 1 (4 points). Let $g(a) = \log_2(3 - a)$.

(a) Find a formula for $g^{-1}(a)$.

Let $b = g(a) = \log_2(3-a)$

Solve for input variable ("a"):

$$b = \log_2(3-a)$$
$$2^b = 2^{\log_2(3-a)}$$
$$2^b = 3-a$$
$$a = 3-2^b \longrightarrow g^{-1}(b) = 3-2^b$$

i.e. $g^{-1}(a) = 3-2^a$

(b) What is the domain and range of $g^{-1}(a)$? Write your answers in inequality or interval notation.

Domain = all possible inputs. $3-2^a$ is defined for all values of a, so domain is $(-\infty, \infty)$.

Range = all possible outputs. I know that $2^a > 0$ for any value of a so $-2^a < 0$ always. Thus $3-2^a$ must be < 3 always.

Range is $(-\infty, 3)$

Problem 2 (4 points). Let $h(y) = 5 \cdot 4^{y+2} - 1$.

(a) Find a formula for $h^{-1}(y)$.

Let $z = h(y) = 5 \cdot 4^{y+2} - 1$. Solve for y (the input variable).

$z = 5 \cdot 4^{y+2} - 1$

$\Rightarrow z+1 = 5 \cdot 4^{y+2}$

$\Rightarrow \frac{z+1}{5} = 4^{y+2}$

$\Rightarrow \log_4\left(\frac{z+1}{5}\right) = \log_4(4^{y+2})$

$\Rightarrow \log_4\left(\frac{z+1}{5}\right) = y+2$

$\Rightarrow y = \log_4(z+1) - \log_4(5) - 2$

So

$h^{-1}(z) = \log_4(z+1) - (\log_4(5)+2)$

or equivalently,

$h^{-1}(y) = \log_4(y+1) - (\log_4(5)+2)$

(b) What is the domain and range of $h^{-1}(y)$? Write your answers in inequality or interval notation.

Domain = All values for which $h^{-1}(y)$ is defined. Since log of a zero or negative number is undefined, need $y+1>0$. So $y>-1$ is the domain.

The range is the set of all outputs of $h^{-1}(y)$. I know the range of $\log_b(x)$ is all real numbers. So $\log_4(y+1)$ outputs all values from $-\infty$ to ∞. This means I can conclude that the outputs of $h^{-1}(y)$ is all real numbers: $\boxed{(-\infty, \infty)}$

Problem 3 (1 point). Suppose $b(u)$ has a single horizontal intercept at $(-\frac{1}{2}, 0)$. If $c(u)$ is the inverse of $b(u)$ (that is, $c(u) = b^{-1}(u)$), where is the vertical intercept of $c(u)$?

If $(\frac{-1}{2}, 0)$ is a point on the graph of $b(u)$, then $b(\frac{-1}{2}) = 0$. ~~b(...)~~. Thus $b^{-1}(0) = \frac{-1}{2}$.

But that means that $\underline{(0, \frac{-1}{2})}$ is the vertical intercept of $b^{-1}(u)$. ~~which~~ And $c(u) = b^{-1}(u)$, so this is the V. Int. of $c(u)$.

Problem 4 (4 points). Let $r(t) = 6^t$. Describe the following functions in terms of transformations of $r(t)$ (for instance, "the function $w(t)$ is $r(t)$ reflected horizontally," or " $w(t)$ is $r(t)$ stretched vertically by a factor of 2.").

(a) $p(t) = 6^t - 4$

p(t) is r(t) shifted down by 4 units

(b) $q(t) = 6^{t+2}$

q(t) is r(t) shifted left by 2 units

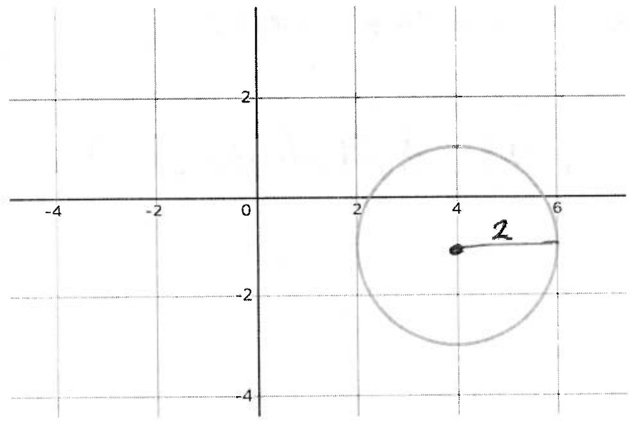
(c) $k(t) = 6^{-t}$

k(t) is r(t) reflected horizontally

(d) $l(t) = -6^t$

l(t) is r(t) reflected vertically

Problem 5 (2 points). Determine the equation of the following graph.



$6-4=2$

Center: $(4,-1)$

Radius: 2

$r^2=(x-h)^2+(y-k^2) \longrightarrow \boxed{4=(x-4)^2+(x+1)^2}$

Problem 6 (2 points). What is the equation of the circle that is centered at the point $(-2, -\frac{4}{3})$ and has radius $\frac{1}{2}$?

$$r^2=(x-h)^2+(y-k)^2$$

$$\left(\frac{1}{2}\right)^2=(x-(-2))^2+\left(y-\left(\frac{-4}{3}\right)\right)^2$$

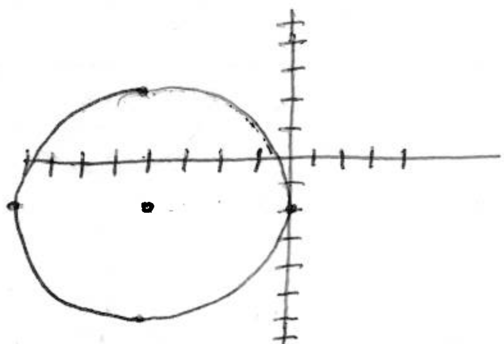
$$\boxed{\frac{1}{4}=(x+2)^2+\left(y+\frac{4}{3}\right)^2}$$

Problem 7 (2 points). Sketch the circle defined by the equation $(x+4)^2+(y+2)^2 = 16$. Make sure your axes have descriptive tick marks so that the center and radius are clear.

$(x+4)^2+(y+2)^2=16$

$r^2 \rightarrow r=4$

center $=(-4,-2)$



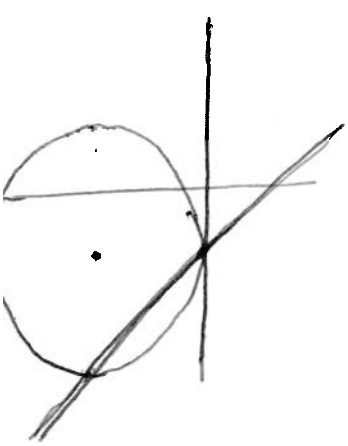
Problem 8 (4 points). Where does the line $f(x) = x - 2$ intersect the circle from Problem 7? Solve for these points algebraically and show your work.

If a point intersects the line and the circle, then its (x,y) coordinates must satisfy both equations. That is,

$$y=x-2 \quad \text{and}$$
$$(x+4)^2+(y+2)^2=16$$

Since $y=x-2$, I can substitute into second equation:

$$(x+4)^2+((x-2)+2)^2=16$$
$$(x+4)^2+x^2=16$$
$$x^2+8x+\cancel{16}+x^2=\cancel{16}$$
$$x^2+8x+x^2=0$$
$$2x^2+8x=0$$
$$x^2+4x=0$$
$$x(x+4)=0$$



$x=0$ or $x=-4$

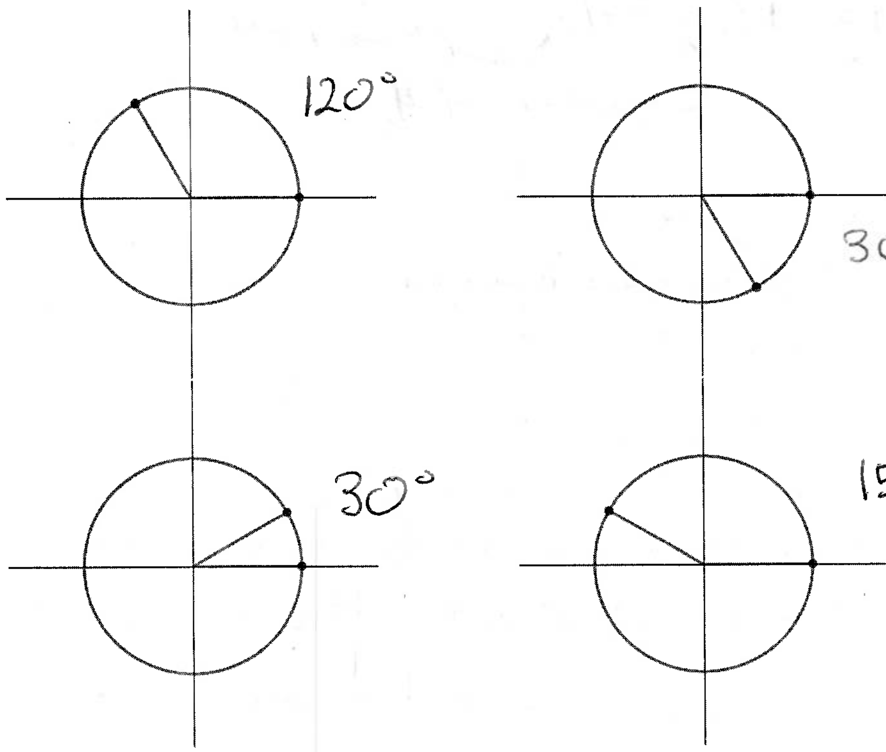
Since $y=x-2$, the y-values for each solution are:

$y=0-2$
$=-2$

$y=-4-2$
$=-6$

Answer: The line and the circle intersect at $(0,-2)$ and $(-4,-6)$

Problem 9 (4 points). Estimate the measure of the following angles. Possible answers are 30, 120, 150, and 300 degrees.



Problem 10 (3 points). Convert the following angle measures into radians. Show your work.

$$\text{Radians} = \frac{\text{Degrees}}{360} \cdot 2\pi$$

(a) 45° $\quad \frac{45}{360} \cdot 2\pi = \frac{1}{8} \cdot 2\pi = \frac{\pi}{4}$

(b) 240° $\quad \frac{240}{360} \cdot 2\pi = \frac{2}{3} \cdot 2\pi = \frac{4\pi}{3}$

(c) −60° $\quad \frac{-60}{360} \cdot 2\pi = \frac{-1}{6} \cdot 2\pi = \frac{-\pi}{3}$

Problem 11 (4 points). Convert the following angle measures from radians into degrees. Show your work.

$$\text{Degrees} = \frac{\text{Radians}}{2\pi} \cdot 360^\circ$$

(a) 2π

$$\frac{2\pi}{2\pi} \cdot 360^\circ = 360^\circ$$

(b) $\frac{\pi}{2}$

$$\frac{\frac{\pi}{2}}{2\pi} \cdot 360^\circ = \frac{\pi}{2} \cdot \frac{1}{2\pi} \cdot 360^\circ = \frac{1}{4} \cdot 360^\circ = 90^\circ$$

(c) $\frac{5\pi}{6}$

$$\frac{\frac{5\pi}{6}}{2\pi} \cdot 360^\circ = \frac{5\pi}{6} \cdot \frac{1}{2\pi} \cdot 360^\circ = \frac{5\pi}{12\pi} \cdot 360^\circ = \frac{5}{12} \cdot 360^\circ = 150^\circ$$

(d) $\frac{11\pi}{6}$

$$\frac{\frac{11\pi}{6}}{2\pi} \cdot 360^\circ = \frac{11\pi}{6} \cdot \frac{1}{2\pi} \cdot 360^\circ = \frac{11}{12} \cdot 360^\circ = 330^\circ$$

Problem 12 (4 points). Determine if the following angles are coterminal. Show your work.

Two angles are coterminal if they differ by a (whole) multiple of 360° or 2π radians

(a) -200° and 160°

$160^\circ - (-200^\circ) = 360^\circ \rightarrow$ multiple of $360^\circ \rightarrow$ coterminal

(b) 30° and -30°

$30^\circ - (-30^\circ) = 60^\circ \rightarrow$ Not a whole multiple of 360°

$\rightarrow$ Not coterminal

(c) $\frac{2\pi}{5}$ and $\frac{7\pi}{5}$

$\frac{2\pi}{5} - \frac{7\pi}{5} = \frac{-5\pi}{5} = -\pi \rightarrow$ Not a whole multiple of 2π

$\rightarrow$ Not coterminal

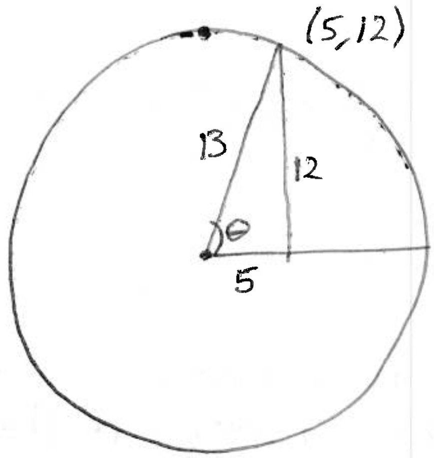
(d) 810° and $\frac{\pi}{2}$

$\frac{\pi}{2}$ to degrees: $\frac{\frac{\pi}{2}}{2\pi} \cdot 360° = \frac{\pi}{2} \cdot \frac{1}{2\pi} \cdot 360° = \frac{1}{4} \cdot 360° = 90°$

Are 810° and 90° coterminal?

$810° - 90° = 720° = 2 \cdot 360° \longrightarrow$ Whole multiple of 360° $\rightarrow$ coterminal

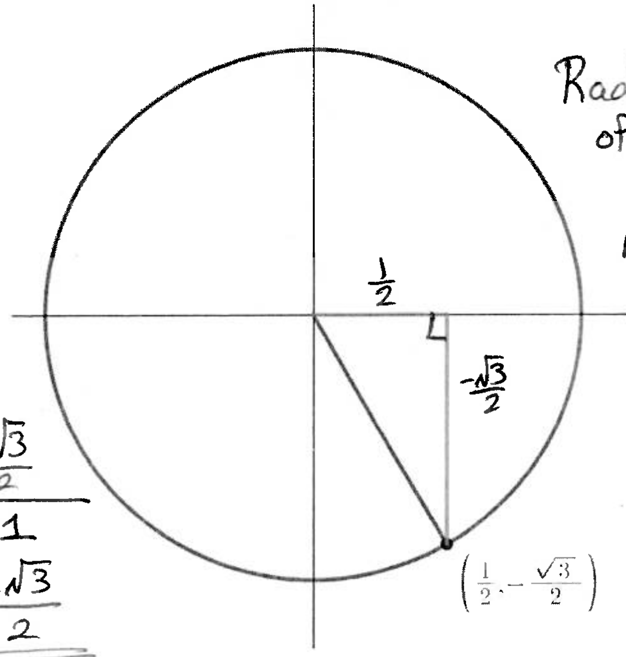
Problem 13 (2 points). The point (5, 12) lies on a circle of radius 13 centered at the origin at an angle θ. Determine the values of $\sin(\theta)$ and $\cos(\theta)$. *Note: you do not need to know what θ is in this problem*



$\sin(\theta) = \frac{y}{r} = \frac{12}{13}$

$\cos(\theta) = \frac{x}{r} = \frac{5}{13}$

Problem 14 (2 points). Let θ be the angle that determines the point in the circle below. Find the values for $\sin(\theta)$ and $\cos(\theta)$.



Radius is the length of the hypotenuse:

$$r^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$

$$r^2 = 1 \Rightarrow \underline{r = 1}$$

$$\sin(\theta) = \frac{y}{r} = \frac{\frac{-\sqrt{3}}{2}}{1} = \underline{\frac{-\sqrt{3}}{2}}$$

$$\cos(\theta) = \frac{x}{r} = \frac{1/2}{1} = \underline{\frac{1}{2}}$$

[OPTIONAL]

Survey Questions.

1. The content in lectures matches well with the content in homework.

Strongly Agree **Agree** **Neither Agree nor Disagree** **Disagree** **Strongly Disagree**

2. The content in lectures matches well with the content in quizzes.

Strongly Agree **Agree** **Neither Agree nor Disagree** **Disagree** **Strongly Disagree**