Name: Solutions PID: ____________

NOTE: You must show the steps necessary to arrive at your answer unless otherwise noted. Use your judgment, if you can't do the entire problem in your head, then you probably should write down at least some intermediate steps.

This assignment has 11 pages. There are 56 total points.

Problem 1 (2 points). You may use a calculator, but you must show your work.

(a) The crow population at UCSD was 3000 at the beginning of 2019 and grows at an annual rate of 9% per year. Write down a formula which outputs the total crow population at UCSD t years after 2019.

$$C(t) = 3000(1.09)^t$$

(b) The UCSD student body has a population of 35,000 people. Assuming that student enrollment stays the same every year, how long do we have until the crows outnumber the students? (That is, what is the first year that the crow population will exceed 35,000?) *[calculator allowed]*

OPTION 1

Using the formula above,

$C(1) = 3270$

$C(5) \approx 4615$

$C(10) \approx 7102$

$C(20) \approx 16813$

$C(30) \approx 39803$

$C(29) \approx 36516$ ← 35000

$C(28) \approx 33501$

OPTION 2

Using logs, I try to find t such that

$C(t) = 3000(1.09)^t = 35000$

$\Rightarrow (1.09)^t = \frac{35000}{3000}$

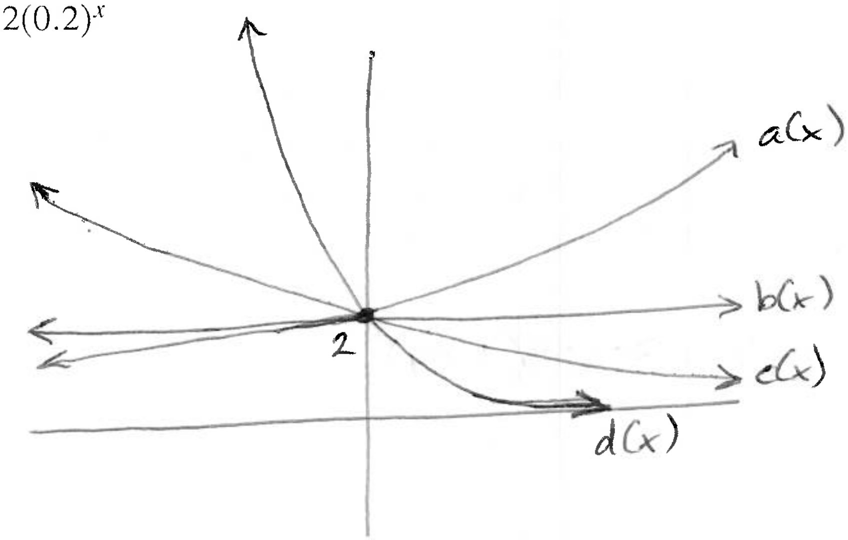
$\Rightarrow \log_{1.09}(1.09^t) = \log_{1.09}\left(\frac{35000}{3000}\right)$

$\Rightarrow t \approx 28.51$

Between 28 and 29 years from 2019, so 2047-2048

Problem 2 (4 points). Make a general sketch of the following functions on the **same** coordinate grid. Do not use a calculator, and do not plot specific points other than the y-intercept (simply make sure that the y-intercept and general shape are correct).

- $a(x) = 2(1.2)^x$
- $b(x) = 2(1.0)^x$
- $c(x) = 2(0.8)^x$
- $d(x) = 2(0.2)^x$



Problem 3 (2 points). Simplify the expression $\frac{x^{r+d}}{x^d} \cdot y^r$ as much as possible.

$$\frac{x^{r+d}}{x^d} \cdot y^r = x^{(r+d)-d} \cdot y^r$$

$$= x^r \cdot y^r = (xy)^r$$

Problem 4 (8 points). Let $r(y) = 3 \cdot 4^y$. Write down a formula for the following functions (remember what you know about transformations of functions!)

(a) $r(y)$ reflected across the vertical axis.

horizontal reflection

$$r(-y) = \boxed{3 \cdot 4^{-y}}$$

$$= 3 \cdot \left(\frac{1}{4}\right)^y$$

optional simplification

(b) $r(y)$ reflected across the horizontal axis.

across horizontal axis = vertical reflection

$$-r(y) = \boxed{-3 \cdot 4^y}$$

(c) $r(y)$ shifted down by two units.

$$r(y) - 2 = \boxed{3 \cdot 4^y - 2}$$

(d) $r(y)$ vertically stretched (expanded) by a factor of two.

$$2 \cdot r(y) = 2 \cdot (3 \cdot 4^y) = \boxed{6 \cdot 4^y}$$

Problem 5 (6 points). Let $s(u) = 3(2)^u$. Write down a formula for the following functions in the form $a \cdot b^u$ (i.e. the exponent is only "u").

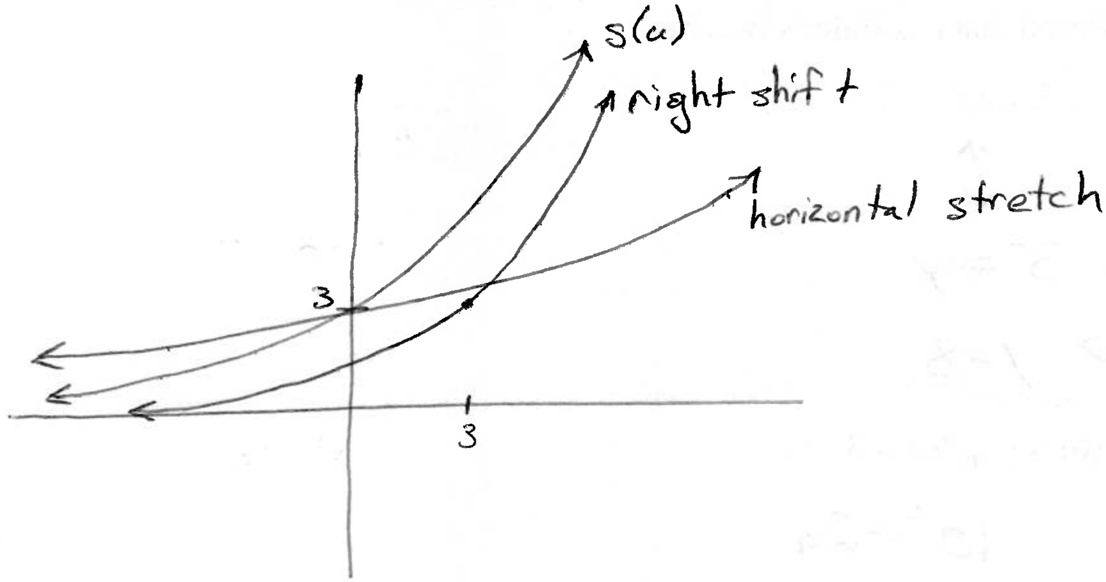
(a) $s(u)$ shifted right by 3 units.

$$s(u-3) = 3 \cdot 2^{u-3}$$
$$= 3 \cdot 2^u \cdot 2^{-3}$$
$$= 3 \cdot 2^u \cdot \frac{1}{8}$$
$$= \underline{\underline{\frac{3}{8} \cdot 2^u}}$$

(b) $s(u)$ horizontally stretched (expanded) by a factor of two.

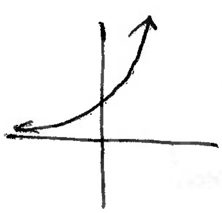
$$s\left(\tfrac{1}{2}u\right) = 3 \cdot 2^{\frac{1}{2}u}$$
$$= 3 \cdot \left(2^{\frac{1}{2}}\right)^u$$
$$= \underline{\underline{3 \cdot \left(\sqrt{2}\right)^u}}$$

(c) Sketch $s(u)$ and the functions from part (a) and (b) on the same coordinate grid.



Problem 6 (4 points). What is the long run behavior of:

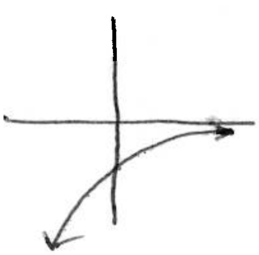
(a) $h(a) = 12(0.3)^a$?



CORRECTION

$h(a)$ tends to 0 as a tends to ∞, and
$h(a)$ tends to ∞ as a tends to $-\infty$.

(b) $c(t) = -0.5 \cdot (0.8)^t$?



$c(t)$ tends to 0 as t tends to ∞, and
$c(t)$ tends to $-\infty$ as t tends to $-\infty$

Problem 7 (4 points). Solve each equation for the variable without using a calculator. Recall that e is Euler's Number.

(a) $\log_3(y) = 2$

$\updownarrow$

$3^2 = y$

$\Rightarrow$ $y = 8$

(b) $\log_{10}(2a) = 3$

$10^3 = 2a$

$1000 = 2a$

$500 = a$

(c) $\ln(x) = 3$

$\ln(x) = \log_e(x)$, so

$e^3 = x$ $\leftarrow$ No need to write this as decimal.

(d) $\log_{16}(x) = \frac{-1}{2}$

$16^{-\frac{1}{2}} = x$

$16^{-\frac{1}{2}} = \left(\frac{1}{16}\right)^{\frac{1}{2}} = \sqrt{\frac{1}{16}} = \frac{\sqrt{1}}{\sqrt{16}} = \frac{1}{4}$, so $x = \frac{1}{4}$

Problem 8 (5 points). Simplify the following expressions:

$\dfrac{2^{3a}}{8^b}$

$2^{3a} = (2^3)^a = 8^a$, so

$$\frac{2^{3a}}{8^b} = \frac{8^a}{8^b} = \underline{\underline{8^{a-b}}}$$

$y^{2\log_y(4)}$

$$y^{2\log_y(4)} = \left(y^{\log_y(4)}\right)^2 = 4^2 = 16$$

Inverse property

$\log_2(2^{10a})$

$\log_2(2^{10a})$

$= 10a$ by inverse property

$\log_{10}(1000)$

$1000 = 10^3$, so

$$\log_{10}(1000) = \log_{10}(10^3) = \underline{\underline{3}}$$

$\log_y(-y)$

Not Graded, typo in problem

~~If $y \le 0$, not defined~~

~~If $y > 0$, $\log_y(-y)$~~

Problem 9 (2 points). Suppose $0 < a < b$. Is $\ln(a)$ less than, greater than, or equal to $\ln(b)$, or is it impossible to tell? Why?

$\ln(a)$ is an increasing function, so
$\ln(b)$ is greater than $\ln(a)$ (since $b > a$).

Problem 10 (1 point). Does the function $g(y) = \ln(y)$ have a horizontal intercept (i.e. a zero)? If so where is it? Write your answer as a coordinate pair.

Yes. $\ln(y)$ is the same as $\log_e(y)$, and every log function has a horizontal intercept at $(1,0)$.

other solution

Find y s.t. $\ln(y) = 0$. If $\ln(y) = 0$, then $e^0 = y$, so $y = 1$.
Thus intercept is at $(1,0)$.

Problem 11 (2 points). Solve the equation $\log_3(x) + \log_3(6x) - \log_3(3x) = 2$ for x by using properties of logarithms to simplify the expression.

$$\log_3(x) + \log_3(6x) - \log_3(3x)$$
$$= \log_3(6x \cdot x) - \log_3(3x)$$
$$= \log_3\left(\frac{6x \cdot x}{3x}\right)$$
$$= \log_3(2x)$$

So then
$\log_3(x) + \log_3(6x) - \log_3(3x) = 2$
$\Rightarrow \log_3(2x) = 2$
$\Rightarrow 3^2 = 2x$
$\Rightarrow 9 = 2x$
$\Rightarrow \underline{x = \frac{9}{2}}$

Problem 12 (2 points). Solve the equation $\ln(\sqrt{\frac{x^3}{8}}) = 3$ by first simplifying the expression using properties of logarithms. Your answer should have an e in it (do not write as a decimal).

$$3 = \ln\left(\sqrt{\frac{x^3}{8}}\right) = \ln\left(\left(\frac{x^3}{8}\right)^{\frac{1}{2}}\right) = \frac{1}{2}\ln\left(\frac{x^3}{8}\right) = \frac{1}{2}\left(\ln(x^3) - \ln(8)\right)$$

So if $3 = \frac{1}{2}(\ln(x^3) - \ln(8))$, then

$6 = \ln(x^3) - \ln(8)$

$\Rightarrow 6 + \ln(8) = \ln(x^3)$

$\Rightarrow e^{6+\ln(8)} = e^{\ln(x^3)}$

$\Rightarrow e^{6+\ln(8)} = x^3$

$\Rightarrow e^6 \cdot e^{\ln(8)} = x^3$

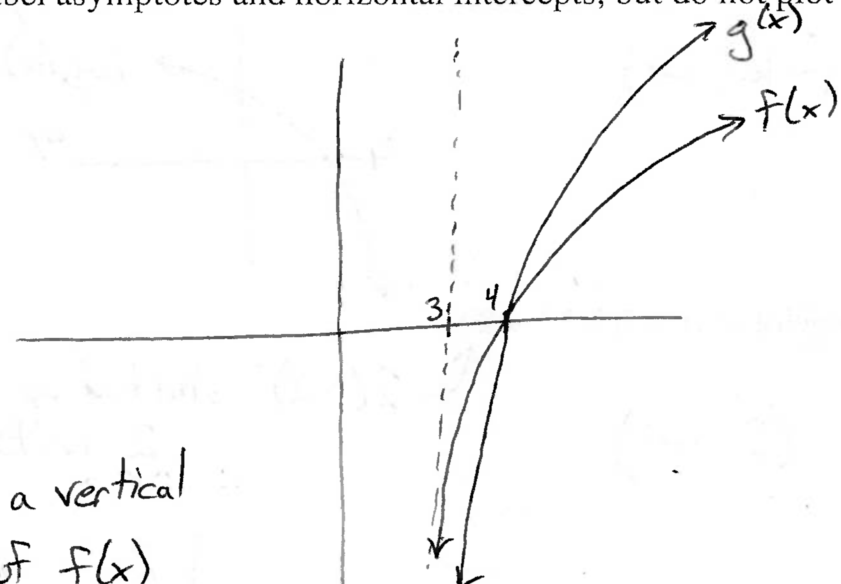
$\rightarrow e^6 \cdot 8 = x^3$

$\Rightarrow \sqrt[3]{e^6 \cdot 8} = \sqrt[3]{x^3}$

$\Rightarrow \sqrt[3]{e^6} \cdot \sqrt[3]{8} = x$

$\Rightarrow e^2 \cdot 2 = x \rightarrow \underline{\underline{x = 2e^2}}$

Problem 13 (2 points). Let c be some number between 2 and 5. You do not know what the exact value is, but you will use it in this problem.
Roughly sketch $f(x) = \log_{10}(x-3)$ and $g(x) = c \cdot \log_{10}(x-3)$ on the same coordinate plane. Label asymptotes and horizontal intercepts, but do not plot any other points.



g(x) is a vertical stretch of f(x)

Problem 14 (8 points).

(a) What is the domain of $f(y) = \log_{10}(y)$?

$(0, \infty)$

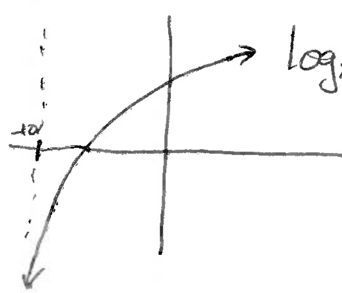
(b) What is the range of $f(y) = \log_{10}(y)$?

$(-\infty, \infty)$

inputs

(c) Let x be some number greater than 1. What is the $\boxed{\text{domain}}$ of $k(a) = \log_x(a+10)$?

$(-10, \infty)$



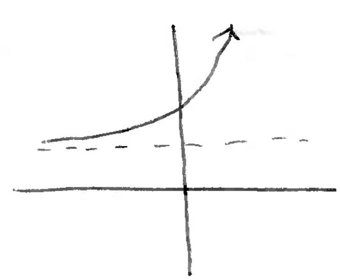
$\log_x(a)$ shifted left by 10 units

outputs

(d) What is the $\boxed{\text{range}}$ of $a(s) = 2(5.2)^s + 2$?

$(2, \infty)$

$2(5.2)^s$ shifted up by 2 units

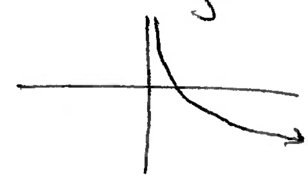


Problem 15 (4 points).

(a) Let b be some positive number bigger than 1. Is $f(y) = \log_b(\frac{1}{y})$ increasing, decreasing, or partially increasing and partially decreasing? You may want to simplify using properties of logarithms.

$$\log_b(\tfrac{1}{y}) = \log_b(y^{-1}) = -1 \cdot \log_b(y)$$

This is a vertical reflection of $\log_b(y)$



Since $\log_b(y)$ is increasing this function must be decreasing.

(b) What is the domain of $f(y)$?

$(0, \infty)$

(c) What is the range of $f(y)$?

$(-\infty, \infty)$

[OPTIONAL]

Survey Questions.

1. Do you find the lectures to go:

too fast **too slow** **roughly the right speed**

2. What is one course topic that you feel most confident in? What is one you feel least sure about?