Name: Solutions PID: ____________

NOTE: You must show the steps necessary to arrive at your answer unless otherwise noted. Use your judgment, if you can't do the entire problem in your head, then you probably should write down at least some intermediate steps.

This assignment has 12 pages. There are 70 total points.

Problem 1 (2 points). Let $p(x) = -x^2 + 8x - 15$.

(a) Factor $-x^2 + 8x - 15$.

$$-x^2+8x-15 = -(x^2-8x+15)$$
$$= -(x-5)(x-3)$$

(b) What are the roots of $p(x)$?

$x=5$ and $x=3$

Problem 2 (4 points). Let $h(y) = 2(y-2)^2(y+1)^2(y+3)$.

(a) What are the roots of $h(y)$?

$y=2$
$y=-1$
$y=-3$

(b) Compute the values of $h(-4)$, $h(-2)$, $h(0)$, and $h(3)$.

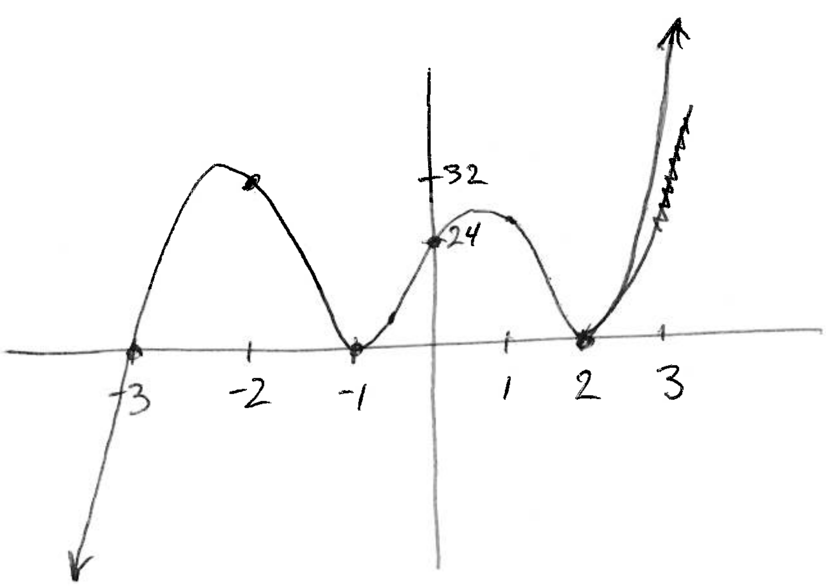
$h(-4) = 2(-4-2)^2(-4+1)^2(-4+3)$
$= 2\cdot(-6)^2\cdot(-3)^2\cdot(-1) = -648$

$h(-2) = 2(-2-2)^2(-2+1)^2(-2+3) = 2\cdot(-4)^2\cdot(-1)^2\cdot 1 = 32$

$h(0) = 2(0-2)^2(0+1)^2(0+3) = 2\cdot(-2)^2\cdot 1^2\cdot 3 = 24$

$h(3) = 2(3-2)^2(3+1)^2(3+3) = 2\cdot 1^2\cdot 4^2\cdot 6 = 192$

(c) Sketch the graph of $h(y)$ using what you know from parts (a) and (b).



Problem 3 (4 points). Let $q(z) = 4(z+5)^2(z+2)^2(z-1)(z-4)^5$.

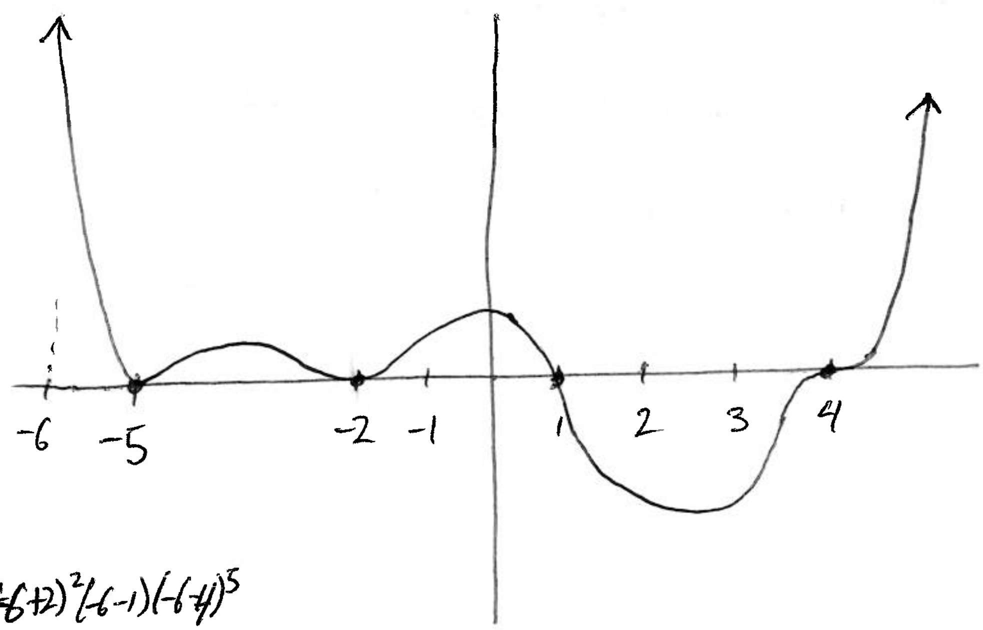
(a) What are roots of $q(z)$?

$z = -5$
$z = -2$
$z = 1$
$z = 4$

(b) What are the multiplicities of each of these roots?

$z = -5 \longrightarrow 2$
$z = -2 \longrightarrow 2$
$z = 1 \longrightarrow 1$
$z = 4 \longrightarrow 5$

(c) Sketch a graph of $q(z)$.



$q(-6) = 4\cdot(-6+5)^2(-6+2)^2(-6-1)(-6-4)^5$
$= 4\cdot(-1)^2(-4)^2(-7)(-10)^5$
> 0

so $\boxed{q(z) \to \infty \text{ as } z \to -\infty}$

Problem 4 (4 points). What are the vertical asymptotes of $g(c) = \frac{2c}{(c-1)(c+2)}$? Your answers should be equations of lines.

Check where denominator is zero, and numerator isn't.

$(c-1)(c+2) = 0$

$\Rightarrow c = 1$ or $c = -2$

Numerator not zero for either of these values, so

The vertical asymptotes are the lines $c = 1$ and $c = -2$

Problem 5 (4 points). What are the vertical asymptotes of $r(b) = \frac{b-1}{(2b+1)(b-1)(b+1)}$? Your answers should be equations of lines.

$(2b+1)(b-1)(b+1) = 0$

$\Rightarrow b = \frac{-1}{2}$ or $b = 1$ or $b = -1$

Do any of these make numerator zero?

Yes, if $b=1$, then numerator $=0$.

So the vertical asymptotes are the lines $b = \frac{-1}{2}$ and $b = -1$

Problem 6 (2 points). The function $f(x) = \frac{2x+1}{(x-2)(2x+1)}$ has a "hole" in it. What are the coordinates of this hole?

A hole is where numerator and denominator are both zero.

Set $2x+1=0 \Rightarrow x=\frac{-1}{2}$

$(x-2)(2x+1)=0 \Rightarrow x=2$ or $x=\frac{-1}{2}$

$x=\frac{-1}{2}$ appears in both!

Hole at $\boxed{x=\frac{-1}{2}}$. To find y-coordinate, cancel: $\frac{2x+1}{(x-2)(2x+1)} = \frac{1}{x-2}$

and plug in $x=\frac{-1}{2}$: $\frac{1}{\frac{-1}{2}-2} = \frac{1}{-5/2} = \boxed{\frac{-2}{5}}$. $\boxed{\text{Answer: } \left(\frac{-1}{2}, \frac{-2}{5}\right)}$

Problem 7 (4 points). Does the function $g(a) = \frac{a^2-4}{a+5}$ have any horizontal asymptotes? If so, where? Justify your answer.

Horizontal asymptotes are: ~~there~~

i) $y=0$ if degree of numerator < degree of denominator

ii) None if degree of numerator > degree of denominator

iii) Ratio of leading coefficients if degrees equal.

In this case, deg. of numerator is 2, deg. of denominator is 1, so $\boxed{\text{no horizontal asymptotes}}$

Problem 8 (4 points). Does the function $g(a) = \frac{a+5}{a^2-4}$ have any horizontal asymptotes? If so, where? Justify your answer.

Deg. of numerator = 1

Deg. of denominator = 2

$\Rightarrow$ H. Asymptote at $y=0$

Problem 9 (10 points). Let $h(a) = \frac{a+1}{2a-1}$.

(a) What is the vertical intercept of $h(a)$? Write your answer as a coordinate pair.

$$h(0) = \frac{0+1}{2\cdot 0 - 1} = \frac{1}{-1} = -1$$

$$\boxed{(0,-1)}$$

(b) What is/are the horizontal intercept(s) of $h(a)$? Write your answer(s) as a coordinate pair.

Check where numerator is zero but denominator isn't:

$a+1=0 \Rightarrow a=-1$

Denominator not zero at $a=-1$ ✓.

So horizontal intercept at $\boxed{(-1,0)}$

(c) Write the equation of the line which is the horizontal asymptote of $h(a)$.

Degrees equal in numerator and denominator.

Ratio of leading coefficients: $\frac{1}{2}$.

$\boxed{\text{Horizontal asymptote at } y=\frac{1}{2}}$

(d) Write the equation of the line which is the vertical asymptote of $h(a)$.

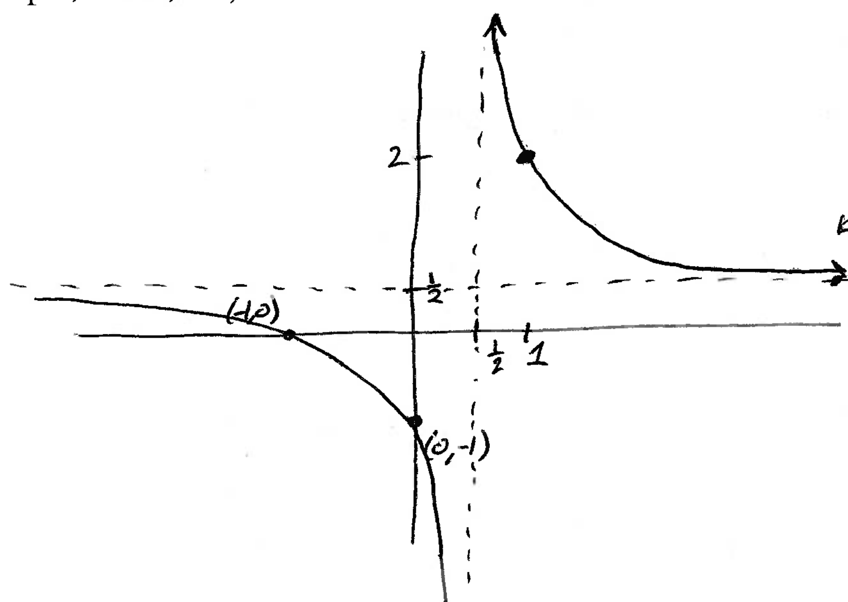
Denominator zero:

$2a-1=0 \Rightarrow a=\frac{1}{2}$.

Check that $a=\frac{1}{2}$ doesn't make numerator 0 ✓

So $a=\frac{1}{2}$ is the vertical asymptote

(e) Sketch a graph of $h(a)$. Label all relevant features of the graph (asymptotes, intercepts, holes, etc).



Point Test:

$h(1) = \frac{1+1}{2\cdot1-1} = \frac{2}{1} = 2$

So graph must be in upper ~~left~~ right corner too.

Problem 10 (10 points). Let $g(b) = \frac{b^2-3b+2}{b^2-1}$.

(a) What is the vertical intercept of ~~$h(a)$~~ $g(b)$? Write your answer as a coordinate pair.

$$g(0) = \frac{0^2-3\cdot0+2}{0^2-1} = \frac{2}{-1} = -2$$

(b) What is/are the horizontal intercept(s) of ~~$h(a)$~~ $g(b)$? Write your answer(s) as a coordinate pair. *(Hint: factor the numerator first.)*

$$b^2-3b+2=0$$
$$(b-2)(b-1)=0$$
$$\Rightarrow b=2 \text{ or } b=1$$

Check if these make ~~numerator~~ denominator 0:

$b=2$: $2^2-1=4-1=3 \neq 0$

$b=1$: $1^2-1=1-1=0$ ⟵ So $b=1$ not an intercept

Therefore, only horizontal intercept is at $b=2$, or $\boxed{(2,0)}$

(c) Write the equation of the line which is the horizontal asymptote of $\cancel{h(a)}$ $g(b)$.

Degrees are equal.

Ratio of leading coefficients is $\frac{1}{1} = 1$

Therefore, horizontal asymptote at $\underline{y=1}$.

(d) Write the equation of the line(s) which is/are the vertical asymptote(s) of $\cancel{h(a)}$ $g(b)$. *(Hint: factor the denominator first.)*

Set $b^2 - 1 = 0$

$\Rightarrow (b-1)(b+1) = 0$

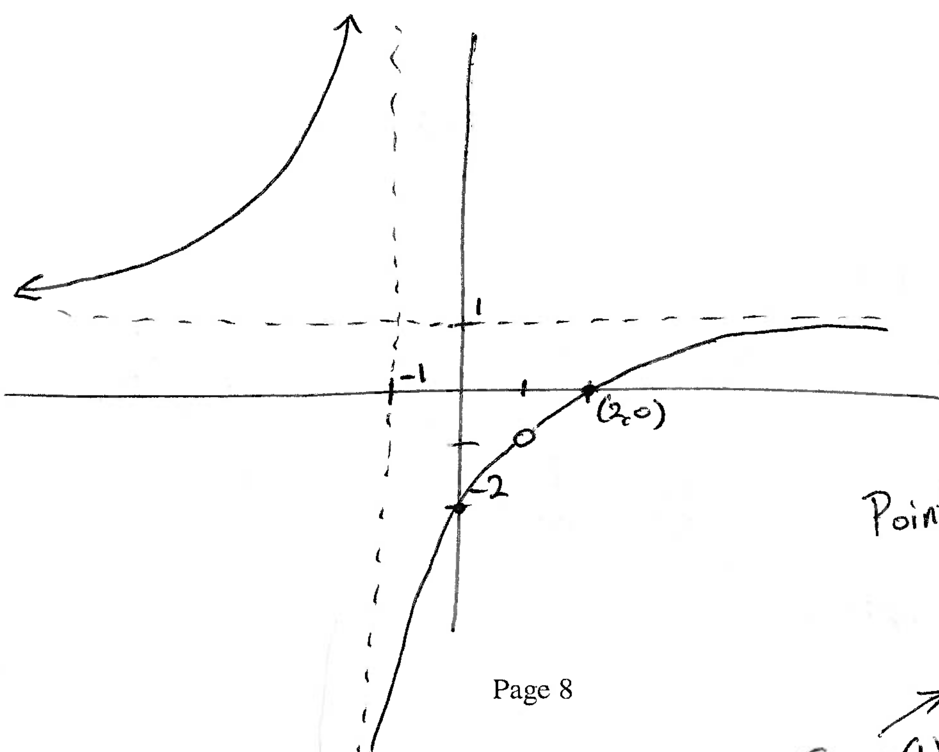
$\Rightarrow b = 1$ or $b = -1$

From (b), we know the numerator is 0 at $b=1$, so there is no asymptote there.

Vertical asymptote at $b=-1$.

(e) Sketch a graph of $\cancel{h(a)}$ $g(b)$. Label all relevant features of the graph (asymptotes, intercepts, holes, etc).

We've seen there is a hole at $\underline{\underline{b=1}}$.



Point Test:

$$g(-2) = \frac{(-2)^2 + (-3)(-2) + 2}{(-2)^2 - 1}$$

$$= \frac{4+6+2}{4-1} = \frac{12}{3} = 4$$

So $g(b)$ is in upper left corner as well

Could also infer this b/c the asymptote $b=-1$ has odd multiplicity

Problem 11 (6 points). Let $g(x) = \frac{x-2}{x-2}$.

(a) Does $g(x)$ have any horizontal asymptotes? If so, what are they?

Yes, $y = \frac{1}{1} = 1$, since degree of numerator and denominator are equal.

(b) Does $g(x)$ have any vertical asymptotes? If so, what are they?

Check where denominator is zero, but numerator isn't.

Denominator $= x-2=0$

$\Rightarrow x=2$.

But $x=2$ also makes numerator zero, so there are <u>no vertical asymptotes</u>

(c) Does $g(x)$ have any holes? If so, what are they?

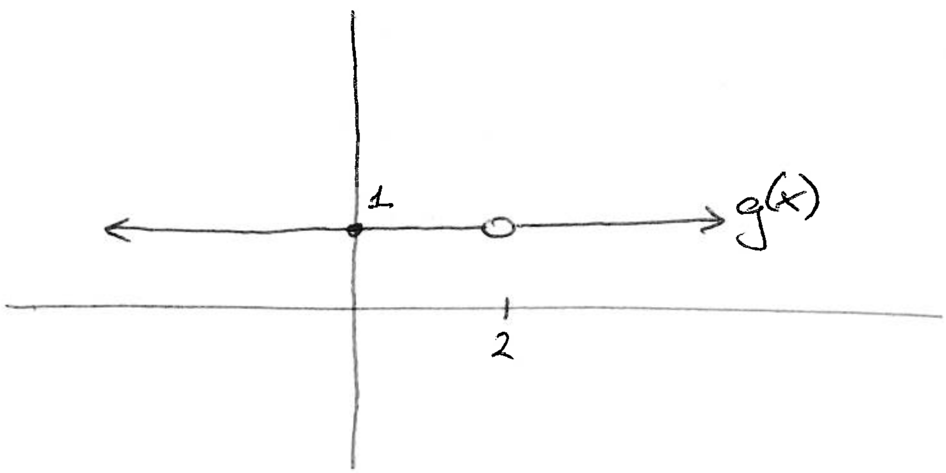
Holes are where both numerator and denominator are zero, so in this case, <u>$x=2$.</u>

To find y-coordinate, ~~sin~~ cancel common factor in $g(x)$:

$\frac{x-2}{x-2} = 1$, and then plug in $x=2$.

Plugging in $x=2$ to the expression "1" just outputs 1 (it is a constant function. So hole is at $\boxed{(2,1)}$

(d) Sketch a graph of $g(x)$.



Problem 12 (8 points). Suppose you have \$5000 in a bank account that earns interest. Each year, you receive 2% interest, and this interest compounds once per year (annually).

(a) How much money is in the account after 1 year?

2% interest → ×1.02 each year - - - same percent increase each year

$M(t) = 5000(1+.02)^t$

$M(1) = 5000(1.02)^1 = 5000 \cdot 1.02 = \underline{5100}$

(b) How much money is in the account after 2 years?

$5100 \cdot 1.02 = \underline{5202}$

(c) How much money is in the account after 3 years?

$5202 \cdot 1.02 = \underline{5306.04}$

(d) Write down a formula, $m(t)$, that outputs the amount of money in the account after t years.

$m(t) = 5000 \cdot 1.02^t$

Problem 13 (8 points). Consider the bank account from the previous problem which begins with a balance of \$5000 dollars. Now, suppose that the account receives 2% interest each year, but that now the interest compounds monthly; that is, you get interest equal to $1/12^{th}$ of the interest rate added each month.

(a) How much money is in the account after 1 month?

2% annual interest $\longrightarrow \frac{2}{12}\%$ interest each month.

$\cancel{5000 \cdot \frac{2}{12}\%}$ $\cancel{5000 \cdot \frac{2}{12} \cdot \frac{1}{100}}$

$\frac{2}{12}\% = \frac{1}{6}\% = \frac{1}{6} \cdot \frac{1}{100} \approx 0.001667$

$5000 \cdot 1.001667 = \underline{5008.33}$

(b) How much money is in the account after 1 year? (you can use a calculator)

$$5000\cdot(1.001667)^{12} = \underline{5100.94}$$

a little more than in Problem 12a!

(c) How much money is in the account after 2 years? (you can use a calculator)

$$5000\cdot(1.001667)^{12\cdot 2} = 5000\cdot 1.001667^{24}$$
$$= \underline{5203.92}$$

also a little more than in Problem 12b!

(d) Write down a formula, $M(t)$, that outputs the amount of money in the account after t years.

$$\cancel{M(t) = 5000\left(1+\frac{0.1667}{12}\right)^{12t}}$$
$$M(t) = 5000\left(1+\frac{.02}{12}\right)^{12\cdot t}$$ or $$5000\cdot(1.001667)^{12\cdot t}$$

[OPTIONAL]

Survey Questions.

1. Roughly how many hours did you spend working on this homework assignment?

2. Have you attended office hours this week, and with whom? *(circle all that apply)*

David **Ashley** **Harveen** **Cathy**