Name: Solutions   PID: _____

> NOTE: You must show the steps necessary to arrive at your
> answer unless otherwise noted. Use your judgment, if you
> can't do the entire problem in your head, then you probably
> should write down at least some intermediate steps.
>
> This assignment has 7 pages. There are 56 total points.

Problem 1 (8 points). Describe the long run behavior of the following functions:

(a) $f(x) = -x^5$

$$f(x) \longrightarrow -\infty \quad \text{as} \quad x \to \infty \quad \text{and}$$
$$f(x) \longrightarrow \infty \quad \text{as} \quad x \to -\infty$$

(b) $h(s) = s^{20}$

$$h(s) \longrightarrow \infty \quad \text{as} \quad s \to \infty \quad \text{and}$$
$$h(s) \longrightarrow \infty \quad \text{as} \quad s \to -\infty$$

(c) $p(z) = -3z^4 + 4z^3 + z^2 - 5z + 5$

$$p(z) \longrightarrow -\infty \quad \text{as} \quad z \to \infty \quad \text{and}$$
$$p(z) \longrightarrow -\infty \quad \text{as} \quad z \to -\infty$$

(d) $q(r) = r^5 + 3r^3 - 4r^6 + 2r$

$$q(r) \longrightarrow -\infty \quad \text{as} \quad r \to \infty \quad \text{and}$$
$$q(r) \longrightarrow -\infty \quad \text{as} \quad r \to -\infty$$

Problem 2 (4 points). Are the following functions even, odd, or neither?

(a) $g(x) = x^5 - x^3 + 2$

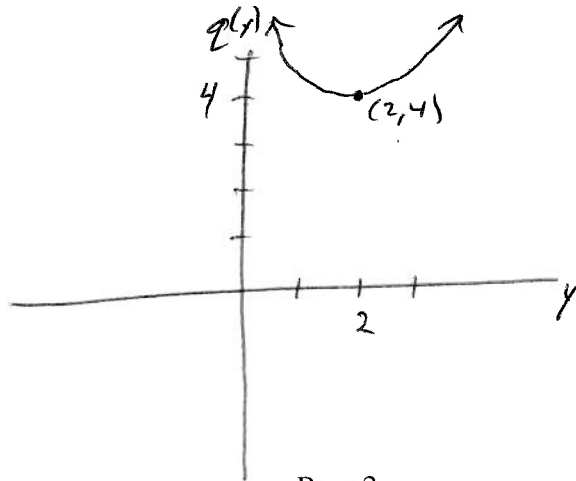$$\text{Neither}$$

(b) $h(a) = a^2 + 5a^6$

$$\text{Even}$$

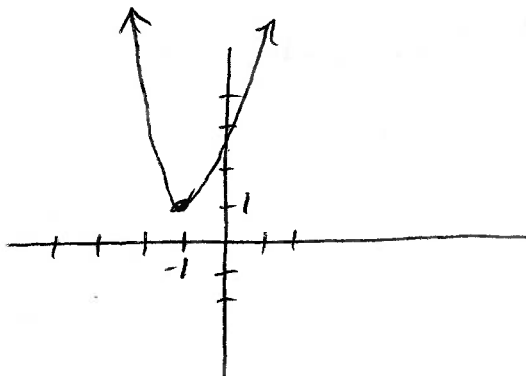(c) $p(x) = x - x^3 - x^7$

$$\text{Odd}$$

(d) $q(x) = 5$

$$\text{Even}$$

Problem 3 (4 points). Sketch the quadratic function $q(y) = 4 + \frac{1}{2}(y-2)^2$. Label your axes.

Problem 4 (4 points). Sketch the quadratic function $w(x) = 2(x + 1)^2 + 1$. Label your axes.

Problem 5 (2 points). Determine the location of the vertex of $r(c) = -5(c + 3)^2 - 4$.

$$(-3, -4)$$

Problem 6 (2 points). Determine the location of the vertex of $w(y) = 3y^2 - 4y + 8$.

Complete the Square:

$h = \frac{2}{3}$, $k = \frac{20}{3}$

So vertex is $\left(\frac{2}{3}, \frac{20}{3}\right)$

Vertex Formula:

$$h = \frac{-b}{2a}$$

$$= \frac{-(-4)}{3 \cdot 2} = \frac{4}{6} = \frac{2}{3}$$

$$k = w\left(\frac{2}{3}\right) = 3 \cdot \left(\frac{2}{3}\right)^2 - 4 \cdot \frac{2}{3} + 8$$

$$= \frac{4}{3} - \frac{8}{3} + \frac{24}{3}$$

$$= \frac{20}{3}$$

Problem 7 (12 points). Factor the following expressions.

(a) $z^2 - 10z + 25$

$$(z-5)^2$$

$(z+a)(z+b)$, where $a+b = -10$
and $a \cdot b = 25$

If $a = -5$ and $b = -5$,
then $-5+-5 = -10$
$(-5) \cdot (-5) = 25$ ✓

(b) $a^2 + a - 6$

$$(a+3)(a-2)$$

$(a+m)(a+n)$, where

$m+n = 1$
$m \cdot n = -6$ $\rightarrow$ $m = 3$
$n = -2$

(c) $a^2 - a - 6$

$$(a-3)(a+2)$$

$(a+m)(a+n)$

where $m+n = -1$ $\rightarrow$ $m = -3$
$m \cdot n = -6$ $n = 2$

(d) $3b^2 + 3b - 6$

$$3(b+2)(b-1)$$

$3b^2 + 3b - 6 = 3(b^2 + b - 2)$

Factor this:

$(b+m)(b+n)$, where

$m+n = 1$
$m \cdot n = -2$ $\rightarrow$ $m = 2$
$n = -1$

Problem 8 (6 points). Write the following quadratic functions in vertex form by completing the square.

(a) $f(x) = x^2 + 4x$

$f(x) = x^2 + 4x$

$f(x) = (x + h)^2 + k$

$\quad = x^2 + 2hx + h^2 + k$

Since $x^2 + 4x = x^2 + 2hx + h^2 + k$

$4 = 2h \longrightarrow h = 2$

$x^2 + 4x = x^2 + 4x + 4 + k$

$\Rightarrow 0 = 4 + k$

$\Rightarrow k = -4$

Therefore $f(x) = (x+2)^2 - 4$

(b) $g(x) = x^2 - 6x + 10$

$g(x) = x^2 - 6x + 10 = (x + h)^2 + k$ ← This is vertex form, so I solve for h and k

$x^2 - 6x + 10 = x^2 + 2hx + h^2 + k$

$-6 = 2h$

$\Rightarrow h = -3$ — Plug h back in

$x^2 - 6x + 10 = x^2 - 6x + 9 + k$

$\Rightarrow 10 = 9 + k$

$\Rightarrow k = 1$

Therefore, $g(x) = (x-3)^2 + 1$

(c) $h(x) = 3x^2 + 12x + 11$

$3x^2 + 12x + 11 = 3(x + h)^2 + k$ ← vertex form

$3x^2 + 12x + 11 = 3(x^2 + 2hx + h^2) + k$

$3x^2 + 12x + 11 = 3x^2 + 6hx + 3h^2 + k$

$12 = 6h$

$\Rightarrow h = 2$ — Plug h back in

$3x^2 + 12x + 11 = 3x^2 + 12x + 3 \cdot 4 + k$

$\Rightarrow 11 = 12 + k$

$\Rightarrow k = -1$

Therefore, $h(x) = 3(x+2)^2 - 1$

Problem 9 (4 points). Find all of the (real) roots of $g(x) = 3x^2 + 6x - 4$ using the quadratic formula. There may be 0, 1, or 2 answers.

$$3x^2 + 6x - 4$$
$$\underset{a}{\uparrow} \quad \underset{b}{\uparrow} \quad \underset{c}{\uparrow}$$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4 \cdot (3) \cdot (-4)}}{2 \cdot 3} = \frac{-6 \pm \sqrt{36 + 48}}{6}$$

$$= \frac{-6 \pm \sqrt{84}}{6} \quad \Rightarrow \quad X = \frac{-6 + \sqrt{84}}{6} \quad \text{or}$$

$$X = \frac{-6 - \sqrt{84}}{6}$$

Problem 10 (4 points). Find all of the (real) roots of $h(x) = 2x^2 - x + 5$. There may be 0, 1, or 2 answers.

$$2x^2 - x + 5$$
$$\underset{a}{\uparrow} \quad \underset{b}{\uparrow} \quad \underset{c}{\uparrow}$$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(5)}}{2 \cdot 2}$$

$$= \frac{1 \pm \sqrt{1 - 40}}{4}$$

$$= \frac{1 \pm \sqrt{-39}}{4} \quad \longleftarrow \quad \sqrt{-39} \text{ is undefined, so}$$

$$\underline{no \ solutions}$$

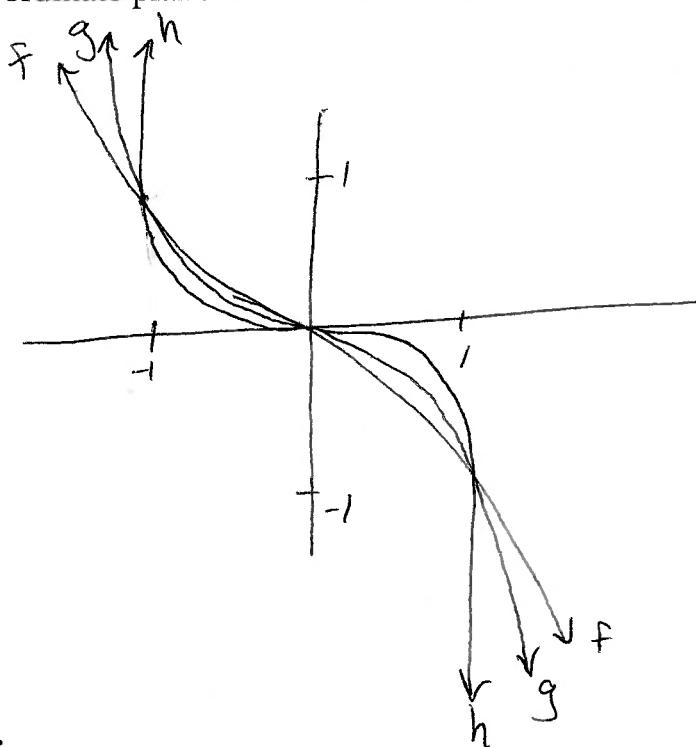**Problem 11** (2 points). What is the vertical intercept of $b(z) = 5(z-3)^2 - 10$?

vertical intercept is point where $z = 0$.

$b(0) = 5(0-3)^2 - 10$
$= 5 \cdot (-3)^2 - 10$
$= 5 \cdot 9 - 10$
$= 45 - 10$
$= 35$

→ vertical intercept is $(0, 35)$

Problem 12 (4 points). Sketch the functions $f(x) = -x^3$, $g(x) = -x^5$, and $h(x) = -x^7$ on the same coordinate plane. Mark the values 1 and -1 on both the vertical and horizontal axes.

[OPTIONAL]
Survey Questions.
1. Roughly how many hours did you spend working on this homework assignment?

2. When you study for this class, do you study... *(circle all that apply)*
    alone   **with a friend**   **with a study group**